MATHEMATICS
GRADE 8

Quality learning @home

DATE: .....

LESSON 1

TOPIC: GEOMETRIC AND NUMERIC PATTERNS - Different kinds of patterns in sequences

CONCEPTS & SKILLS TO BE ACHIEVED:
By the end of the lessons learners should know and be able to:

Investigate and extend patterns

- Investigate and extend numeric and geometric patterns looking for relationships between numbers, including patterns:
  -- represented in physical or diagram form
  -- not limited to sequences involving a constant difference or ratio
  -- of learner's own creation
  -- represented in tables
  -- represented algebraically

- Describe and justify the general rules for observed relationships between numbers in own words or in algebraic language

RESOURCES:	DBE Workbook, Sasol-Inzalo book 1, Textbooks
ONLINE RESOURCES	

LESSON DEVELOPMENT
LESSON 1

INTRODUCTION:

TERMINOLOGY AND SYMBOLS.

- **A number pattern** is a **pattern** or sequence in a series of **numbers**. This **pattern** generally establishes a common relationship between all **numbers**.
- A list of numbers which forms a pattern is called a **sequence.**
- Each number in a sequence is called a **term** of the sequence.
- Numbers that follow one another are said to be **consecutive.**

Examples

$3; 8; 13; 18 \ldots\ldots$

Here, we get the **numbers** in the **pattern** by adding 5 to the previous number each time.

Sequence:

$3; 8; 13; 18 \ldots\ldots\ldots$

$3; 8; 13; 18 \ldots\ldots\ldots$

First term $= 3 \quad (a = T_1 = 3)$

4th Term $= 18 \quad (T_4 = 18)$

- When the difference between consecutive terms of a sequence are the same, we say the difference is **constant.**

$3 \;;\; 8 \;;\; 13 \;;\; 18 \ldots\ldots$
$+5 \quad +5 \quad +5$

The constant difference is 5. $(d = 5)$

$3 \;;\; 1 \;;\; -1 \;;\; -3 \;;\ldots\ldots$
$-2 \quad -2 \quad -2$

The constant difference is -2. $(d = -2)$

$3 \;;\; 8 \;;\; 15 \;;\; 25 \ldots\ldots$
$+5 \quad +7 \quad +9$

Here we add 5, then 7, then 9…
The difference is thus NOT a constant.

- A sequence can be formed by repeatedly multiplying or dividing. In this case the **ratio** between consecutive terms is **constant.**

$3 \;;\; 6 \;;\; 12 \;;\; 24 \;;\ldots\ldots$
$\times 2 \quad \times 2 \quad \times 2$

Here we multiply each term by 2
to get the next term. The constant ratio is:
$\frac{6}{3} = \frac{12}{6} = \frac{24}{12} = 2 \quad (r = 2)$

- A sequence can also be formed in such a way that **neither** the difference nor the ratio between consecutive terms is constant.

$1 \;;\; 1 \;;\; 2 \;;\; 3 \;;\; 5 \ldots\ldots\ldots$
$T_1 + T_2 \quad T_2 + T_3 \quad T_3 + T_4$

Also called Fibonacci pattern

THE TERM-TERM RELATIONSHIP IN A SEQUENCE:

One can identify a pattern or relationship between consecutive terms in order to extend the pattern.

A number pattern can be analysed by focusing on the relationship between consecutive terms as shown in the examples below:

• 3; 7; 11; 15; 19; … starting with 3, 4 is added to get the next term each time

• 1; 2; 4; 7; 11; 16; 22; … starting with 1, add 1 then 2 and then increase the number that is added by 1 each time (so: add 1, 2, 3, 4, …).

CLASSWORK:

WORK THROUGH THE FOLLOWING EXAMPLES THAT WILL EXPLAIN TO YOU HOW TO GO FROM 1 TERM TO THE NEXT. WRITE THE ANSWERS IN YOUR WORKBOOK. ONLY CONSULT THE ANSWERS AT THE END OF THE LESSONS ONCE YOU HAVE COMPLETED IT ON YOUR OWN.

ACTIVITY 1:

1. Write down the next three numbers in each of the sequences below. Also explain in writing, in each case, how you figured out what the numbers should be.

a) Sequence A: 2; 5; 8; 11; 14; 17; 20; 23;
b) Sequence B: 4; 5; 8; 13; 20; 29; 40;
c) Sequence C: 1; 2; 4; 8; 16; 32; 64;
d) Sequence D: 3; 5; 7; 9; 11; 13; 15; 17; 19;
e) Sequence E: 4; 5; 7; 10; 14; 19; 25; 32; 40;
f) Sequence F: 2; 6; 18; 54; 162; 486;
g) Sequence G: 1; 5; 9; 13; 17; 21; 25; 29; 33;
h) Sequence H: 2; 4; 8; 16; 32; 64;

2. Which sequences above are of the same kind as sequence A? Explain your answer.

NB: When the differences between consecutive terms of a sequence are the same, we say the difference is constant.

3. Provide a rule to describe the relationship between the numbers in the sequence. Use this rule to calculate the missing numbers in each sequence.

(a) 1; 8; 15; ; ; ; ; ; ...

(b) 10 020; ; ; ; 9 980; 9 970; ; ; 9 940; 9 930; ...

The same number was added and/or subtracted with each new term

The next example is different to nr 1,2 :

4. Explain how the new term is calculated: 2; 6; 18; 54; 162; 486; ...

Yes, each term is multiplied with 3 every time:

CONSOLIDATION:

- **When the differences between consecutive terms of a sequence are the same, we say the difference is constant.**
- **The number that we multiply with to get the next term in the sequence is called a ratio. If the number we multiply with remains the same throughout the sequence, we say it is a constant ratio**

HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES

1. Copy and complete the table:

Input number	1	2	3	4	5	12	23
Input number + 7	8		10		12		

2. Describe, in words, the rule for finding the next number in the sequences. Write down the next five terms of each sequence.

(a) 1; 10; 100; 1 000; … (b) 16; 8; 4; 2; … (c) 7; −21; 63; −189;…

(d) 3; 12, 48;…

3. Copy and complete the table:

Input number	1	2	3	4		12	n
Output number	6			24	36		

LESSON 2:

INTRODUCTION/ REVISION

Yesterday we learnt that the terms in a sequence can be determined by adding, subtracting and multiplying between terms.

These are sequences where there is a constant difference or a constant ratio between consecutive terms.

CLASSWORK:

TODAY WE WILL INVESTIGATE PATTERNS FORMED BY <u>NEITHER ADDING NOR MULTIPLYING</u> BY THE SAME NUMBER

WORK THROUGH THE FOLLOWING EXAMPLES THAT WILL EXPLAIN TO YOU HOW TO GO FROM 1 TERM TO THE NEXT. WRITE THE ANSWERS IN YOUR WORKBOOK. ONLY CONSULT THE ANSWERS AT THE END OF THE LESSONS ONCE YOU HAVE COMPLETED IT ON YOUR OWN.

ACTIVITY 1:

1. Consider the sequence: 10; 17; 26; 37; 50; ...

(a) Write down the next five numbers in the sequence.

2. Which of the statements below can be used to describe the relationship between the numbers in the sequence? Test the rule for the first three terms of the sequence and then simply write "yes" or "no" next to each statement

(a) Increase the difference between consecutive terms by two each time.

(b) Increase the difference between consecutive terms by one each time.

(c) Add two more than you added to get the previous term.

CONSOLIDATION:

THERE ARE SEQUENCES WHERE THERE IS NEITHER A CONSTANT DIFFERENCE NOR A CONSTANT RATIO BETWEEN CONSECUTIVE TERMS AND YET A PATTERN STILL EXISTS

HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES

1. Provide a rule to describe the relationship between the numbers in the sequences below. Use your rule to provide the next five numbers in each sequence.

(a) 1; 4; 9; 16; 25;

(b) 2; 13; 26; 41; 58;

(c) 4; 14; 29; 49; 74;

(d) 5; 6; 8; 11; 15; 20;

LESSON 3:

INTRODUCTION: THE POSITION-TERM RELATIONSHIP IN A SEQUENCE

In this lesson we will learn how we can use the position of terms to make predictions in extending patterns

LESSON DEVELOPMENT

CLASSWORK:

WORK THROUGH THE FOLLOWING DEVELOPING EXAMPLES. WRITE THE ANSWERS IN YOUR WORKBOOK. ONLY CONSULT THE ANSWERS AT THE END OF THE LESSONS ONCE YOU HAVE COMPLETED IT ON YOUR OWN.

Activity 1:

Consider the following sequence 5; 8; 11; 14; 17; 20; 23;...

Sizwe has been thinking about Amanda and Tamara's explanations of how they worked out the rule for sequence A and has drawn up a table. He agrees with them but says that there is another rule that will also work. He explains:

My table shows the terms in the sequence and the difference between consecutive terms:

1	2	3	4	5	6	7
5	8	11	14	17	20	23

+3	+3	+3	+3	+3	+3

Sizwe reasons that the following rule will also work:

Multiply the position of the number by 3 and add 2 to the answer. I can write this rule as a number sentence: Position of the number × 3 + 2 I use my number sentence to check: **1** *× 3 + 2 = 5;* **2** *× 3 + 2 = 8;* **3** *× 3 + 2 = 11*

1. (a) What do the numbers in bold in Sizwe's number sentence stand for?

   (b) What does the number 3 in Sizwe's number sentence stand for?

2. Consider the sequence 5; 8; 11; 14; ...

   Apply Sizwe's rule to the sequence and determine:

   (a) term number 7 of the sequence

   (b) term number 10 of the sequence

   (c) the hundredth term of the sequence

HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES

1. Consider the sequence: 3; 5; 7; 9; 11; 13; 15; 17; 19; ...

   (a) Use Sizwe's explanation to find a rule for this sequence.

   (b) Determine the 28th term of the sequence.

2. Copy and complete the following tables by calculating the missing terms:

a.

Position in sequence	1	2	3	4	10	54
Term	4	7	10	13		

b.

Position in sequence	1	2	3	4	8	16
Term	4	9	14	19		

c.

Position in sequence	1	2	3	4	7	30
Term	3	15	27			

3. Copy the following table. Use the rule **position in the sequence × (position in the sequence + 1)** to complete it.

Position in sequence	1	2	3	4	5	6
Term	2					

LESSON 4:

INTRODUCTION: INVESTIGATING AND EXTENDING GEOMETRIC PATTERNS

We can represent numbers as an arrangement of dots or circles in various shapes, for example triangles, squares, pentagons, hexagons and so on. These numbers are called polygonal numbers. For example, 15 balls can be arranged as a triangle with five balls in a row, four in a row above them, three in a row above those, two in a row above the three and one in the top row. These numbers are sometimes called figurate numbers.

These polygons are two-dimensional, but we can extend the representation to three and even to higher dimensions.

THIS LESSON WILL FOCUS ON INVESTIGATING AND EXTENDING GEOMETRIC PATTERNS

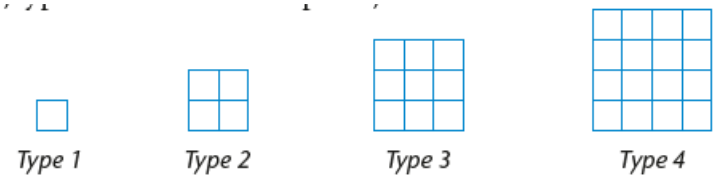
LESSON DEVELOPMENT

CLASSWORK:

WORK THROUGH THE FOLLOWING DEVELOPING EXAMPLES. WRITE THE ANSWERS IN YOUR WORKBOOK. ONLY CONSULT THE ANSWERS AT THE END OF THE LESSONS ONCE YOU HAVE COMPLETED IT ON YOUR OWN.

Activity 1: Square numbers

A factory makes window frames. Type 1 has one windowpane, type 2 has four windowpanes, type 3 has nine windowpanes, and so on.



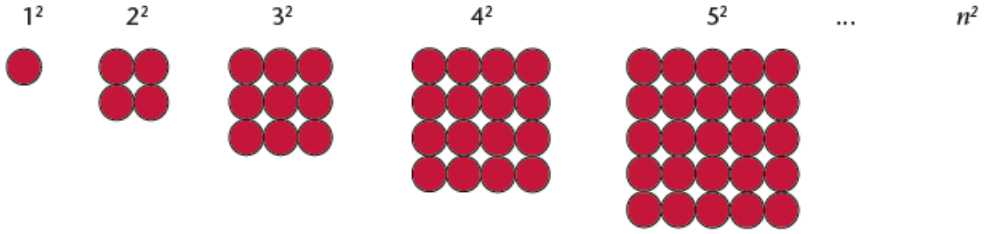
1. How many windowpanes will there be in type 5?
2. How many windowpanes will there be in type 6?
3. How many windowpanes will there be in type 7?
4. How many windowpanes will there be in type 12? Explain.
5. Copy and complete the table. Show your calculations.

Frame type	1	2	3	4	15	30
Number of window panes	1	4	9			

CONSOLIDATION:

In algebra we think of a square as a number that is obtained by multiplying a number by itself. So, 1 is also a square because 1 × 1 = 1.

The symbol *n* is used below to represent the *position number* in the expression that gives the rule (n^2) when generalising.

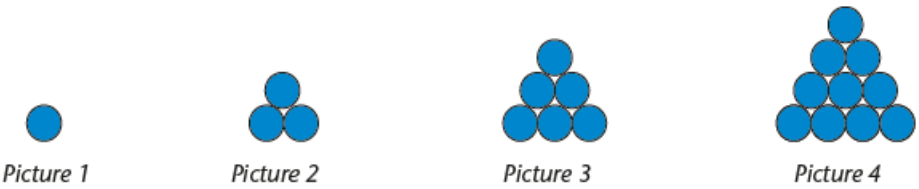


HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES:

TRIANGULAR NUMBERS:

1. Therese uses circles to form a pattern of triangular shapes:



1. If the pattern is continued, how many circles must Therese have:

   (a) in the bottom row of picture 5?

   (b) in the second row from the bottom of picture 5?

   (c) in the third row from the bottom of picture 5?

   (d) in the second row from the top of picture 5?

   (e) in the top row of picture 5?

   (f) in total in picture 5? Show your calculations.

2. How many circles does Therese need to form triangle picture 7? Show the calculation.

3. How many circles does Therese need to form triangle picture 8?

4. Copy and complete the following table. Show all your work.

Picture number	1	2	3	4	5	6	12	15
Number of circles	1	3	6					

LESSON 5:

INTRODUCTION:

Lesson 4 dealt with square numbers and with triangular numbers:

Triangular numbers being the numbers 3, 6, 10, 15 and so on could form a triangular pattern. They represented these numbers with dots which they arranged in such a way that they formed equilateral triangles, hence the name triangular numbers.

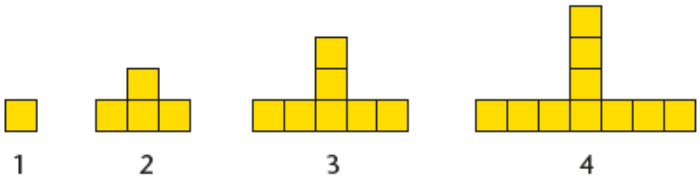
TODAY WE WILL INVESTIGATE DESCRIBING PATTERNS IN DIFFERENT WAYS

LESSON DEVELOPMENT

CLASSWORK:

WORK THROUGH THE FOLLOWING DEVELOPING EXAMPLES. WRITE THE ANSWERS IN YOUR WORKBOOK. ONLY CONSULT THE ANSWERS AT THE END OF THE LESSONS ONCE YOU HAVE COMPLETED IT ON YOUR OWN

The pattern below is made from squares.



1. (a) How many squares will there be in pattern 5?

(b) How many squares will there be in pattern 15?

(c) Copy and complete the following table:

Pattern number	1	2	3	4	5	6	20
Number of squares	1	4	7	10			

You can use the following three plans (or methods) to calculate the number of squares for pattern 20. Study each one carefully.

Plan A:

To get from one square to four squares, you have to add three squares. To get from four squares to seven squares, you have to add three squares. To get from seven squares to ten squares, you have to add three squares. Continue to add three squares for each pattern until pattern 20.

Plan B:

Multiply the pattern number by three and subtract two. Pattern 20 will therefore have 20 × 3 − 2 squares.

Plan C:

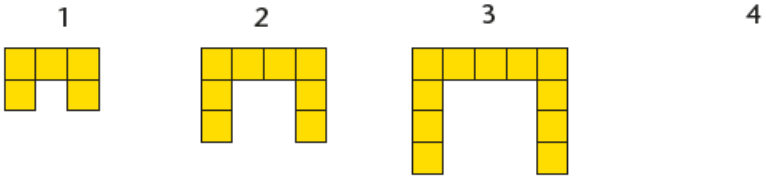
The number of squares in pattern 5 is 13. Pattern 20 will therefore have 13 × 4 = 52 squares because 20 = 5 × 4.

HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES:

Three figures are given below.

1 a) Draw the next figure in the tile pattern.



(b) If the pattern is continued, how many tiles will there be in the 17th figure?

LESSON 6:

INTRODUCTION:

Today we will do some more exercises on patterns.

HOMEWORK:

COMPLETE THE EXERCISES IN YOUR CLASSWORK BOOK. ONLY CONSULT THE MEMORANDUM ONCE YOU HAVE DONE ALL THE EXERCISES:

1. Write down the next four terms in each sequence. Also explain, in each case, how you figured out what the terms are.

(a) 2; 4; 8; 14; 22; 32; 44;

(b) 2; 6; 18; 54; 162;

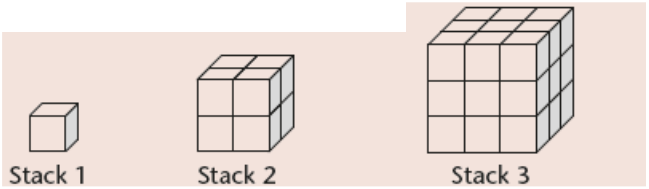
(c) 1; 7; 13; 19; 25;

2. (a) Copy and complete the following table by calculating the missing terms:

Position in sequence	1	2	3	4	5	7	10
term	3	10	17				

(b) Write the rule to calculate the term from the position in the sequence in words.

3. Consider the stacks below



(a) How many cubes will there be in stack 5?

(b) Copy and complete the following table:

Stack number	1	2	3	4	5	6	10
Number of cubes	1	8	27				

(c) Write down the rule to calculate the number of cubes for any stack number.

MEMORANDUM:

LESSON 1:

CLASSWORK:

Activity 1:

1

a) 26; 29; 32 – Add 3 each time.
b) 53; 68; 85 – Add consecutive odd numbers: 1 then 3 then 5 and so on.
c) 128; 256; 512 – Multiply by 2 each time.
d) 21; 23; 25 – Add 2 each time or count in odd numbers.
e) 49; 59; 70 – Add consecutive natural numbers: 1 then 2 then 3 and so on.
f) 1 458; 4 374; 13 122 – Multiply by 3 each time.
g) 37; 41; 45 – Add 4 each time.
h) 128; 256; 512 – Multiply by 2 each time.

2. Sequence D and sequence G. A constant number is added each time.

3. a) 22; 29; 36; 43
b) 10 010; 10 000; 9 990;

Rules:
(a) add 7 (b) subtract 10

HOMEWORK:

1.

Input number	1	2	3	4	5	12	23
Input number + 7	8	9	10	11	12	19	30

2. (a) 10 000; 100 000; 1 000 000; 10 000 000; 100 000 000
Multiply each number by 10 to calculate the next number.
(b) 1; 0,5; 0,25; 0,125; 0,0625
Divide each number by 2 to calculate the next number.
(c) 567; −1 701; 5 103; −15 309; 45 927
Multiply each number by −3 to calculate the next number.
(d) 192; 768; 3 072; 12 288; 49 152
Multiply each term by 4 to calculate the next number.

3.

Input number	1	2	3	4	6	12	*n*
Output number	6	12	18	24	36	72	*n X* 6

LESSON 2:

CLASSWORK:

Activity 1:

1. (a) 65; 82; 101; 122; 145

2. a) Yes. Test: The difference between 17 and 10 is 7. Increase the difference by 2 to get 9. To get the next term, add 9 to 17 to get 26.
   Continue this way of reasoning to get the other terms.
   (b) No. Test: The difference between 17 and 10 is 7. Increase the difference by 1 to get 8. To get the next term add 8 to 17. It gives 25 which is not the next term in the sequence.
   (c Yes. The test is the same as that in question (a).

HOMEWORK:

1. (a) 36; 49; 64; 81; 100 The term value is the square of the term number.
   (b) 77; 98; 121; 146; 173 Add consecutive odd numbers: 11; 13; 15; …
   (c) 104; 139; 179; 224; 274 Add consecutive multiples of 5: 10; 15; 20; …
   (d) 26; 33; 41; 50; 60 Add consecutive natural numbers: 1; 2; 3; …

LESSON 3:

CLASSWORK:

Activity 1:

1. (a) The term number or the position of the number in the sequence.
   (b) It is the constant difference between the terms

2. (a) $7 \times 3 + 2 = 21 + 2 = 23$
   (b) $10 \times 3 + 2 = 30 + 2 = 32$
   (c) hundredth term = $100 \times 3 + 2 = 300 + 2 = 302$

HOMEWORK:

1. (a) Position of the number × 2 + 1 (or $2n + 1$)
   (b) twenty-eighth term = $28 \times 2 + 1 = 56 + 1 = 57$

2. a) 31; 163
   b)39; 79
   c) 39; 75 ;351

3. 6; 12; 20; 30; 42

LESSON 4:

CLASSWORK:

Activity 1:

1. 25
2. 36
3. 49
4. There will be 144 windowpanes. To find the number of windowpanes you square the type number. So for type 12 there are $12 \times 12 = 144$ windowpanes.
5. $4 \times 4 = 16$; $15 \times 15 = 225$; $30 \times 30 = 900$

HOMEWORK:

1. (a) 5 (b) 4 (c) 3 (d) 2 (e) 1
   (f) 15 circles: 5 + 4 + 3 + 2 + 1 = 15 (add the number of circles in each row)
2. 7 + 6 + 5 + 4 + 3 + 2 + 1 = 28 circles
3. 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 36 circles
4. 10; 15; 21; 78; 120;

LESSON 5:

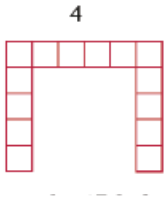
CLASSWORK:

Activity 1:

1. a) 13
   b) 43
   c) 13; 16; 58

HOMEWORK:

1. a)



b) Three tiles are added to make each figure: one tile across and two tiles on the right-hand side downwards. To make the seventeenth figure, three tiles must be added thirteen times (17 – 4 = 13) to the 14 tiles of the fourth figure. So there will be 14 + 13 × 3 = 53 tiles.

LESSON 6:

HOMEWORK:

1 (a) 58; 74; 92; 112
Add consecutive even numbers: 2 then 4 then 6 and so on.
(b) 486; 1 458; 4 374; 13 122
Multiply each number by 3 to find the next term.
(c) 31; 37; 43; 49
Add 6 to find the next term

2 a) 24; 31; 45; 66
b) Multiply the position number by 7 and subtract 4.

3 a) 125
b) 64; 125; 216; 1 000
c) Cube the stack number,