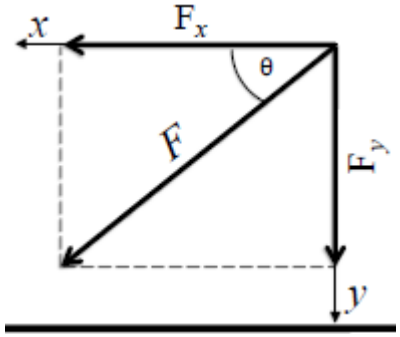
SUBJECT and GRADE	Physical Sciences Grade 11	
TERM 1	*Week 2*	
TOPIC	Vectors in two dimensions	
AIMS OF LESSON	At the end of this lesson you should be able to: • Calculate the magnitude of the resultant using the theorem of Pythagoras. • Determine the direction of the resultant using simple trigonometric ratios • Determine the resultant (R) of two vectors graphically using either the tail-to-head • or tail-to-tail method (parallelogram method) as well as by calculation (component method). • Explain the meaning of a closed vector diagram.	
RESOURCES	***Paper based resources***	***Digital resources***
	Text books, pen, pencil; ruler ;protractor and paper.	See simulation: 23FV at www.everythingscience.co.za See video: 23FW at www.everythingscience.co.za
INTRODUCTION	➢ Revise Gr.10 work on vectors and first week concepts ➢ we will take these concepts further and learn about vectors in two dimensions as well as components of vectors.	
CONCEPTS AND SKILLS	**Key Mathematics Concepts that is key in order to grasp the topic** • Theorem of Pythagoras — Mathematics, Grade 10, Analytical geometry • Units and unit conversions — Physical Sciences, Grade 10, Science skills • Equations — Mathematics, Grade 10, Equations and inequalities • Trigonometry — Mathematics, Grade 10, Trigonometry • Graphs — Mathematics, Grade 10, Functions and graphs	

Vectors in 2D

RESOLVING INTO COMPONENTS

Diagonal vectors can be broken into components. When vectors are broken into the x- and y-component s , we are determining the horizontal (x-axis) and vertical (y-axis) effect of the vector.



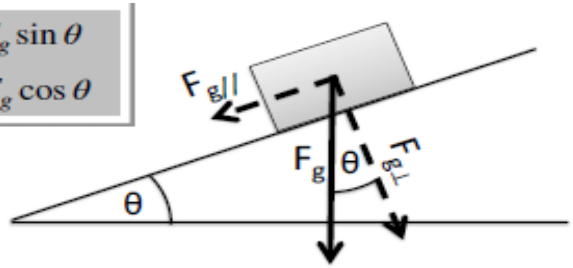
$$F_x = F\cos\theta$$
$$F_y = F\sin\theta$$

COMPONENTS ON A SLOPE

When forces act on objects on a slope, it is useful to resolve vectors into components that are parallel (//) or perpendicular (⊥) components.

The most common force resolved into components on a slope is weight (Fg).

$$F_{g//} = F_g\sin\theta$$
$$F_{g\perp} = F_g\cos\theta$$

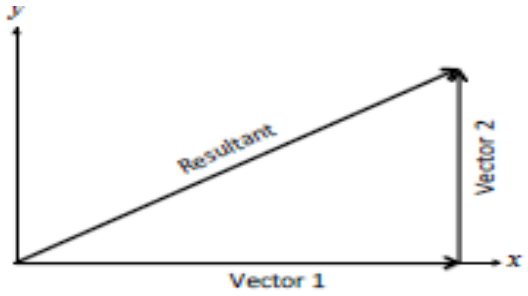


<u>Finding the magnitude of the resultant in two dimensions graphically:</u>

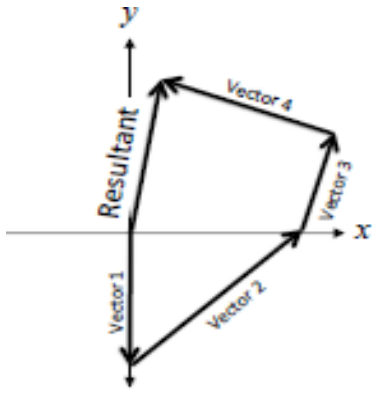
Step 1: Choose a scale and draw axes
Step 2: Draw *Rx*
Step 3: Draw *Ry*
Step 4: Draw the resultant vector, *R*

Tail-to-head Method

Used for consecutive vectors (vectors that occur in sequence). Eg. A boat travels 90 m east, and then moves 50 m north.

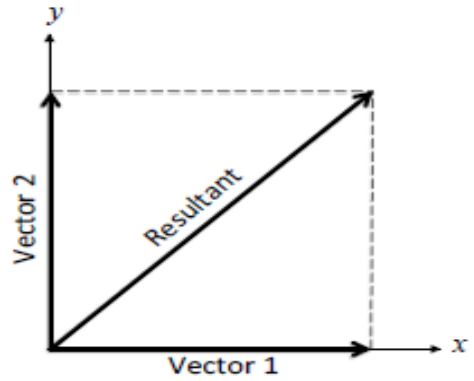


This principle can also be applied to more than 2 vectors taken in order. The resultant is from the tail of the first vector to the head of the last

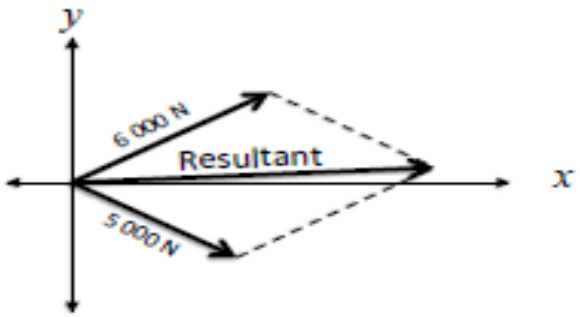


Parallelogram

Used for vectors that act **concurrently** on the same object. The resultant is the **diagonal** of a **parallelogram** that originates from the tail of the vectors.



Eg. Two tugboats apply a force of 6 000N and 5 000N at bearings of 60° and 120° respectively on a cargo ship.



PYTHAGORAS (90° ONLY)

Pythagoras can only be applied to vector triangles that are right angle triangles.

FOR FINDING SIDES:

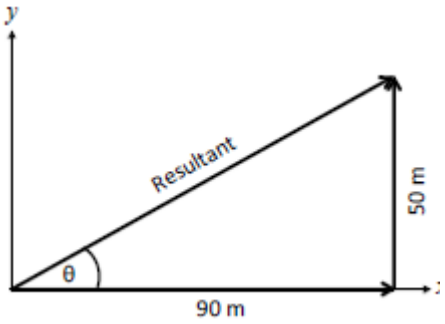
$$R^2 = x^2 + y^2$$

FOR FINDING ANGLES:

$$\sin\theta = \frac{o}{h} \quad \cos\theta = \frac{a}{h} \quad \tan\theta = \frac{o}{a}$$

EXAMPLE:

A boat travels 90 m due east, and then moves 50 m due north. Determine the displacement of the boat.



$R2 = x2 + y2$

$\sqrt{R = 902 + 502}$

R = 102,96 m

tan θ = o/a

θ = tan−1(50/90)

θ = 29,05∘

Remember that θ calculated is relative to the x-axis,

∴ bearing = 90∘ − 29,05∘ = **60**, **95**∘

∴ Displacement = 102,96 m at a bearing of 60,95∘

COMPONENT ADDITION

The resultant of diagonal forces can be determined using Pythagoras by determining the x-resultant and y-resultant first. This is especially useful for determining resultants when more than 2 forces act on an object and a force triangle cannot be used.

Steps to determine resultant using component method:

1.Determine the x- and y-components of each force.
2.Determine the x- and y-resultants of components.
3.Determine the resultant using Pythagoras.
4.Determine the angle using trigonometric principles

EXAMPLE:
Three forces act on an object as shown in the diagram below. Determine the resultant force on the object.

STEP 1 : DETERMINE THE X-AND Y COMPONENT OF EACH FORCE

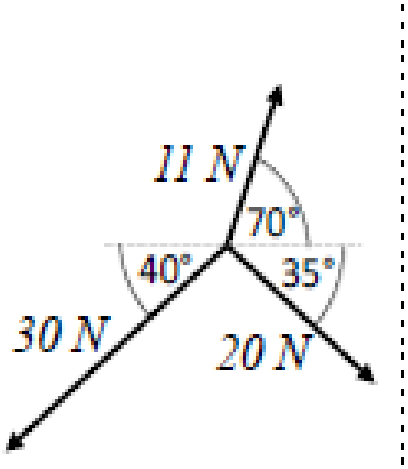
1. 11N force:

$Fx = F \cos \theta$
$= 11 \cos 70$
$= 3,76$ N right

$Fy = F \sin \theta$
$= 11 \sin 70$
$= 10,34$ N up

30N force:

$Fx = F \cos \theta$
$= 30 \cos 40$
$= 22,98$ N left
$Fy = F \sin \theta$
$= 30 \sin 40$
$= 19,28$ N down



20N force:

$Fx = F \cos \theta$

= 20 cos 35

= 16,38 N right

$Fy = F \sin \theta$

= 20 sin 35

= 11,47 N down

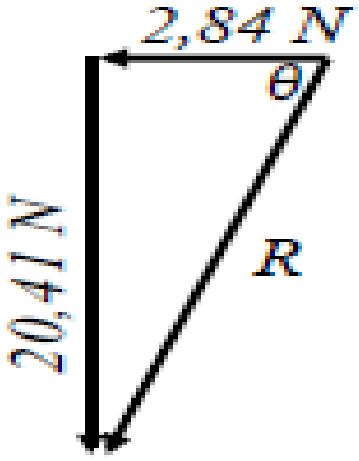
STEP 2. DETERMINE THE x- and y-resultant

$Fx = -3,76 + 22,98 - 16,38$

= 2,84 N left

$Fy = -10,34 + 19,28 + 11,47$

= 20,41 N down



STEP 3.CALCULATE THE Resultant & 4. Angle

$R2 = x2 + y2$

$R = 2,842 + 20,412$

$R = 20,61$ N

$\tan \theta = oa$

$\theta = \tan{-1}\ 20,41$

2,84

$\theta = 82,08°$

∴Resultant = 20,61 N at a bearing of 187,92°

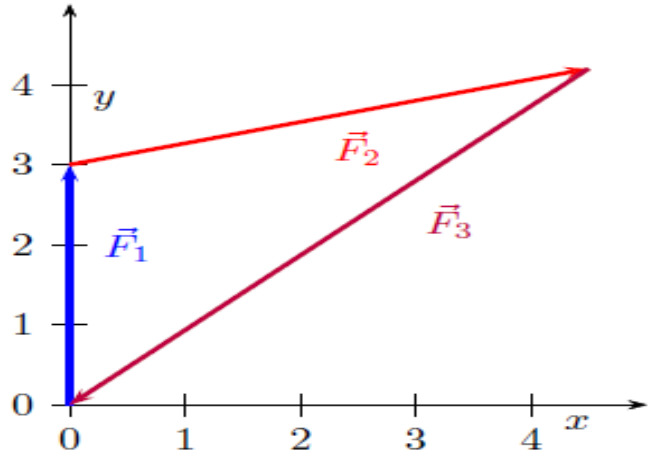
(The equilibrant will be 20,61 N at a bearing of 7,92°)

Closed vector diagrams

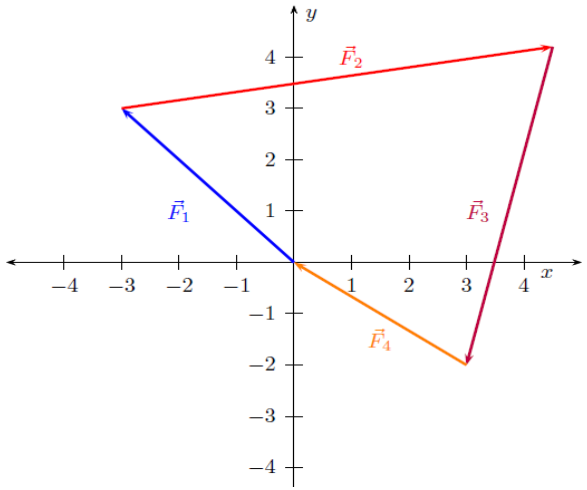
- A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero.
- This means that if the first vector starts at the origin the last vector drawn must end at the origin. The vectors form a closed polygon, no matter how many of them are drawn.

- Here are a few examples of closed vector diagrams:

EXAMPLE



- In this case there were 3 force vectors. When drawn tail-to-head with the first force starting at the origin the last force drawn ends at the origin.
- The resultant would have a magnitude of zero. The resultant is drawn from the tail of the first vector to the head of the final vector.

	2. In the diagram below there are 4 vectors that also form a closed vector diagram  ➢ In this case with 4 vectors, the shape is a 4-sided polygon. ➢ Any polygon made up of vectors drawn tail-to-head will be a closed vector diagram because a polygon has no gaps.
ACTIVITIES/ ASSESSMENT	**ACTIVITY 1** 1.Find the resultant in the x-direction, *Rx*, and *y*-direction, *Ry* for the following forces: *a)* *F*1 = 4,8 N in the positive x-direction *b)* *F*2 = 3,2 N in the negative x-direction *c)* *F*3 = 1,9 N in the positive *y*-direction *d)* *F*4 = 2,1 N in the negative *y*-direction 2.Sketch the resultant of the following force vectors using the tail-to-head method: a) _*F*1 = 4,8 N in the positive *y*-direction b) *F*2 = 3,3 N in the negative x-direction **ACTIVITY** 3.Sketch the resultant of the following force vectors using the tail-to-head method by first determining the resultant in the x- and *y*-directions:

	a) *F*1 = 5,2 N in the positive *y*-direction b) *F*2 = 7,5 N in the negative *y*-direction c) *F*3 = 4,8 N in the positive *y*-direction d) *F*4 = 6,3 N in the negative x-direction 4. Four forces act simultaneously at a point, find the resultant if the forces are: a) *F*1 = 2,3 N in the positive x-direction b) *F*2 = 4,9 N in the positive *y*-direction c) *F*3 = 4,3 N in the negative *y*-direction d) *F*4 = 3,1 N in the negative *y*-direction 13. Resolve each of the following vectors into components: a) *F*1=105 N at 23,5_ to the positive x-axis. b) *F*2=27 N at 58,9_ to the positive x-axis. c) *F*3=11,3 N at 323_ to the positive x-axis.
CONSOLIDATION	**SUMMARY** ➢ Vectors can be added graphically using the head-to-tail method or the tail-to-tail method. ➢ A closed vector diagram is a set of vectors drawn on the Cartesian using the tail-to-head method and that has a resultant with a magnitude of zero. ➢ Vectors can be added algebraically using Pythagoras' theorem or using components. ➢ The direction of a vector can be found using simple trigonometric calculations. ➢ The components of a vector are a series of vectors that, when combined, give the original vector as their resultant. ➢ Components are usually created that align with the Cartesian coordinate axes. ➢ For a vector *F* that makes an angle of with the positive *x*-axis the *x*-component is $Rx = R\cos(\theta)$ and the *y*-component is $Ry = R\sin(\theta)$.
VALUES	*Different actions in your daily life will have more than one outcome. Your actions impact on other people in the system.* *You can apply the principle of vectors in sport, workplace and in the medical field when you do eye tests et.* *Accurate measurement is an important skill in vectors.*