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| SUBJECT and GRADE | Physical Sciences grade 11 | |
| TERM 1 | *Week 1* | |
| TOPIC | Vectors in two dimension | |
| AIMS OF LESSON | At the end of this lesson you should be able to:<br>• Define a resultant.<br>• Determine the resultant of vectors (maximum four) on a Cartesian plane, using the component method.<br>• Sketch the vertical vector ($R_y$) and the horizontal vector ($R_x$) on a Cartesian plane. | |
| RESOURCES | ***Paper based resources*** | ***Digital resources*** |
| | *Text books, pen, pencil; ruler ;protractor and paper.* | See simulation: 23FV at www.everythingscience.co.za<br>See video: 23FW at www.everythingscience.co.za |
| INTRODUCTION | Revise gr.10 work on vectors.<br>We often use arrows to represent vectors visually because the length of the arrow can be related to the magnitude and the arrowhead can indicate the direction. In grade 10 you learnt about vectors in one dimension. Now we will take these concepts further and learn about vectors in two dimensions as well as components of vectors. | |
| CONCEPTS AND SKILLS | *It's important to understand and remember the following concepts learned in gr.10 highlighted in bold. Make use of other resources and textbook for more examples to broaden understanding.*<br>**Scalar:** a physical quantity that has magnitude only<br>**Vector**: a physical quantity that has both magnitude and direction<br>**Resultant vector**: the single vector which has the same effect as the original vectors acting together<br>**Distance**: the length of path travelled (scalar quantity)<br>**Displacement:** a change in position (vector quantity)<br>**Speed:** the rate of change of distance (scalar quantity)<br>**Velocity:** the rate of change of displacement (vector quantity)<br>**Acceleration:** the rate of change of velocity (vector quantity) | |

In **grade 10** you learnt about the resultant vector in one dimension, we are going to extend this to two dimensions.:

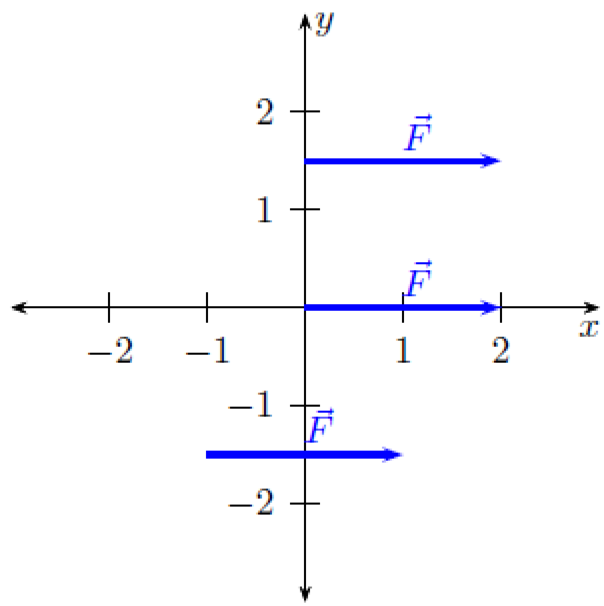
RESULTANT: The resultant vector will have the same effect as all the vectors adding together.

**FINDING THE RESULTANT OF VECTORS IN TWO DIMENSION**

The first thing to make a note of is that in Grade 10 we worked with vectors all acting in a line, on a single axis. In two dimensions it can be represented:

- by using the Cartesian plane which consists of two perpendicular (at a right angle) axes.
- We normally draw the x-axis from left to right (horizontally) and the *y*-axis up and down (vertically).

Example




In the diagram the vectors have:

- The same magnitude
- because the arrows are the same length
- they have the same direction.

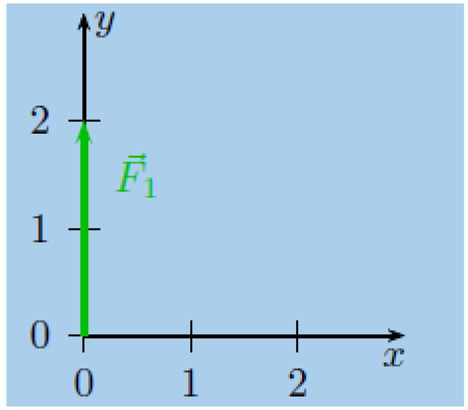
They are all parallel to the x-direction and parallel to each other.

**EXAMPLE:**

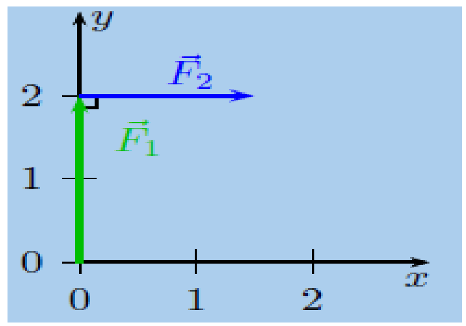
**A)**Sketch the resultant of the following force vectors using the tail-to-head method:

- *F*1 = 2 N in the positive *y*-direction
- *F*2 = 1,5 N in the positive *x*-direction
- *F*3 = 1,3 N in the negative *y*-direction
- *F*4 = 1 N in the negative *x*-direction

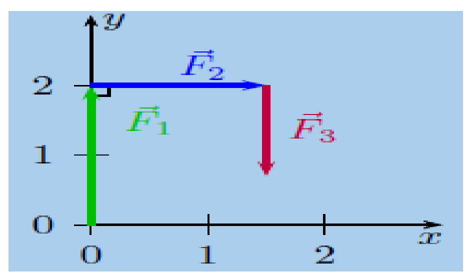
**Step 1: Draw the Cartesian plane and the first vector**



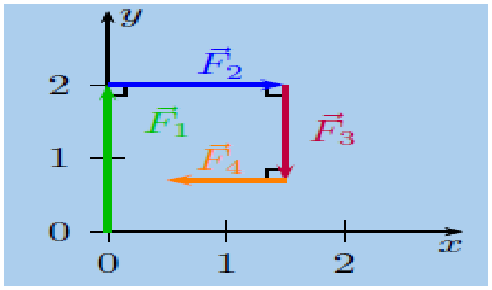

**Step 2: Draw the second vector Starting at the head of the first vector we draw the tail of the second vector:**




**Step 3: Draw the third vector starting at the head of the second vector we draw the tail of the third vector:**

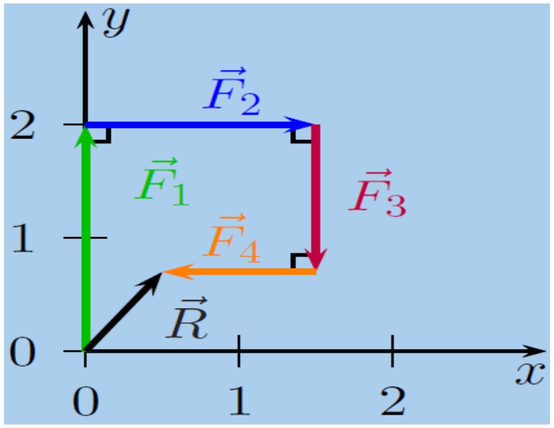


**Step 4: Draw the fourth vector**




**Step 5: Draw the resultant vector**

Starting at the origin draw the resultant vector to the head of the fourth vector:



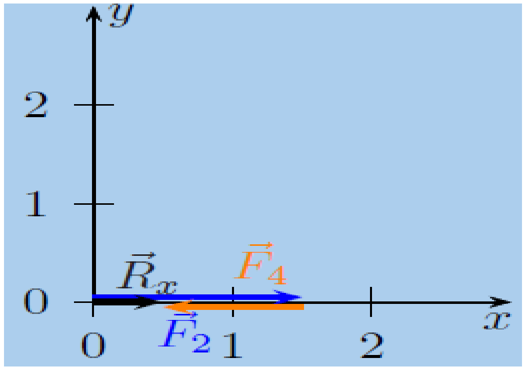

**EXAMPLE:**

**B)** Sketch the **resultant** of the following force vectors using the tail-to-head method by first determining the resultant in the **x- and y-directions**:

- *F*1 = 2 N in the positive *y*-direction
- *F*2 = 1,5 N in the positive *x*-direction
- *F*3 = 1,3 N in the negative *y*-direction
- *F*4 = 1 N in the negative *x*-direction

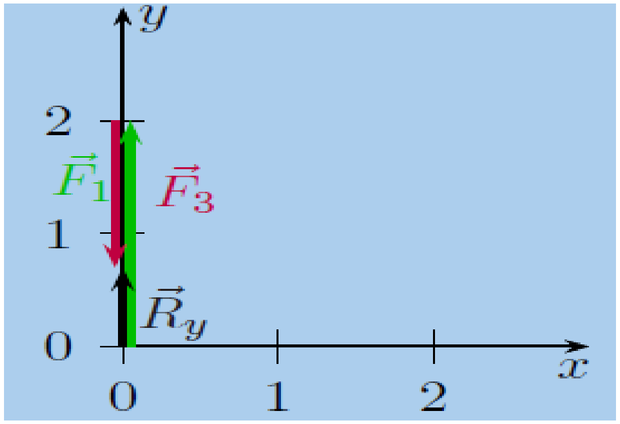
**Step 1: First determine *Rx***

**First draw the Cartesian plane with the vectors in the *x*-direction:**

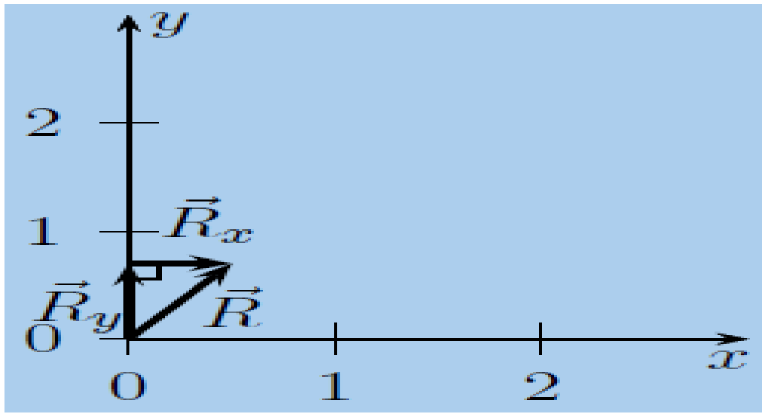



**Step 2: Secondly determine *Ry***

**Next we draw the Cartesian plane with the vectors in the *y*-direction:**



**Step 3: Draw the resultant vectors,**
***Ry* and _*Rx* head-to-tail**

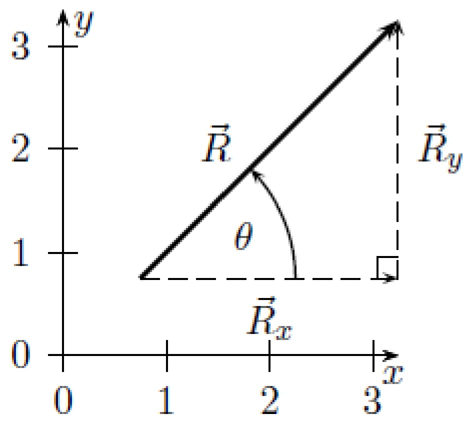



***COMPONENTS OF VECTORS:***

- In the discussion of vector addition, we saw that a number of vectors acting together can be combined to give a single vector (the resultant).
- In much the same way a single vector can be broken down into a number of vectors which when added give that original vector.
- These vectors which sum to the original are called components of the original vector.

In practise it is most useful to resolve a vector into components which are at right angles to one another, usually horizontal and vertical.

Any vector can be resolved into a horizontal and a vertical component. If_***R*** is a vector, then the **horizontal component** of_***R* is_*Rx*** and the **vertical component** is_***Ry*.**




- When resolving into ***components that are parallel to the x- and y-axes*** we are always dealing with a **right-angled triangle.**
- This means that we can use ***trigonometric identities*** to determine the magnitudes of the components (we know the directions because they are aligned with the axes).

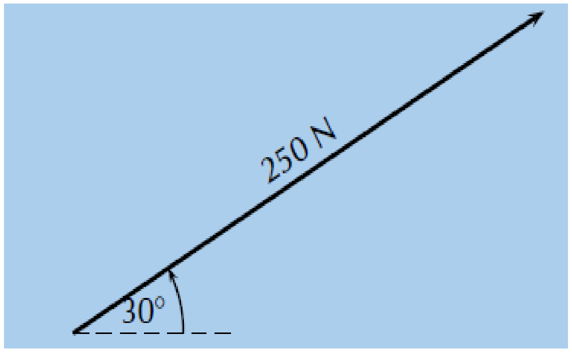
$$R_x = R\cos(\theta)$$

$$R_y = R\sin(\theta)$$

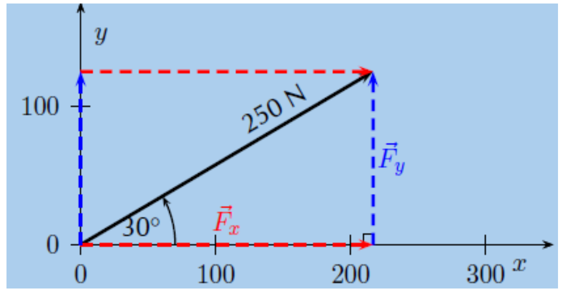
**Example**

A force of 250 N acts at an angle of 30_ to the positive x-axis. Resolve this force into components parallel to the x- and y-axes.

**Step 1**: Draw a rough sketch of the original vector



**Step 2: Determine the vector components**




**Step 3: Determine the magnitudes of the component vectors**

*Fy* = 250 sin(30)= 125 N   and   *Fx* = 250 cos(30) = 216,5 N

***Remember Fx and Fy are the magnitudes of the components. Fx is in the positive x-direction and Fy is in the positive y-direction***

Components can also be used to find the resultant of vectors. This technique can be applied to both graphical and algebraic methods of finding the resultant. The method is straightforward:

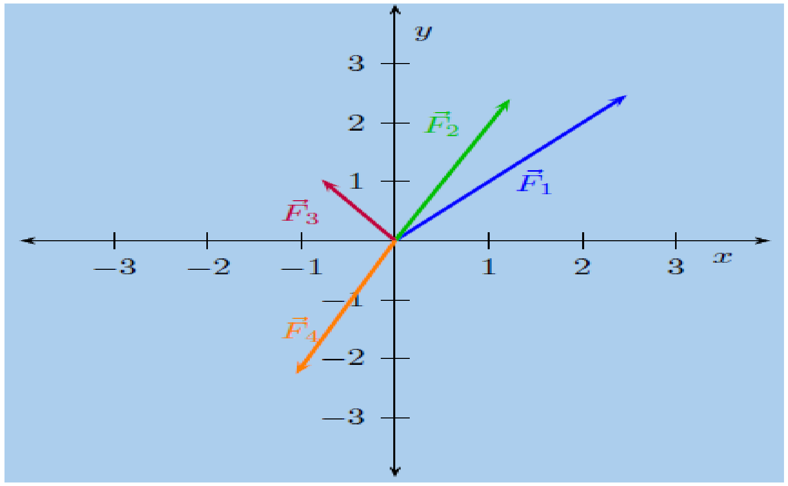
1. make a rough sketch of the problem;
2. find the horizontal and vertical components of each vector;
3. find the sum of all horizontal components, *Rx*;
4. find the sum of all the vertical components, *Ry*;
5. then use them to find the resultant, *R*.

**EXAMPLE**

Determine, by resolving into components, the resultant of the following four forces acting at a point:

- *F*1=3,5 N at 45 to the positive x-axis.
- *F*2=2,7 N at 63 to the positive x-axis.
- *F*3=1,3 N at 127 to the positive x-axis.
- *F*4=2,5 N at 245 to the positive x-axis.

**Step 1: Sketch the problem**




**Step 2: Determine components of *F*1 –F4**

**Firstly we find the magnitude of the vertical component, *F*1y:**

sin(θ) = *F*1*y*/*F*1

sin(45)= *F*1*y*/3,5

*F*1*y* = (sin(45)) (3,5)

= 2,47 N

**magnitude of the horizontal component, *F*1x:**

cos(θ) =*F*1x/*F*1

cos(45_) =*F*1x/3,5

*F*1x = (cos(45_)) (3,5)

= 2,47 N

**magnitude of the vertical component, *F2*y**:

sin(θ) =*F*2*y*/*F*2

sin(63) =*F*2*y*/2,7

*F*2*y* = (sin(63_)) (2,7)

= 2,41 N

**horizontal component, *F*2x**:
cos *(θ)* =*F*2x/*F*2
cos(63) =*F*2x/2,7
*F*2x = (cos(63_)) (2,7)
= 1,23 N

**vertical component, *F*3y:**
sin(θ) =*F*3*y*/*F*3
sin(127) =*F*3*y*/1,3
*F*3*y* = (sin 127) (1,3)
= 1,04 N

**horizontal component, *F*3x:**
cos(θ) =*F*3x/*F*3
cos(127) =*F*3x/1,3
*F*3x = (cos 127) (1,3)
= −0,78 N

**vertical component, *F*4y:**
sin(θ) =*F*4*y*/*F*4
sin(245) =*F*4*y*/2,5
*F*4*y* = (sin(245_)) (2,5)
= −2,27 N

**horizontal component, *F*4x**:
cos(*θ*) =*F*4x/*F*4
cos(245_) =*F*4x/2,5
*F*4x = (cos(245_)) (2,5)
= −1,06 N

**STEP 3 Determine components of resultant**

| Vector | $x$-component | $y$-component | Total |
|---|---|---|---|
| $\vec{F}_1$ | 2,47 N | 2,47 N | 3,5 N |
| $\vec{F}_2$ | 1,23 N | 2,41 N | 2,7 N |
| $\vec{F}_3$ | −0,78 N | 1,04 N | 1,3 N |
| $\vec{F}_4$ | −1,06 N | −2,27 N | 2,5 N |
| $\vec{R}$ | 1,86 N | 3,65 N | |

Use the Theorem of Pythagoras to determine the magnitude of the resultant

$R^2 = (R_y)^2 + (R_x)^2$
$= (1,86)^2 + (3,65)^2$
$= 16,78$
$R = 4,10$ N

We can also determine the angle with the positive x-axis:

$\tan(\theta) = 1,86/3,65$
$\theta = \tan^{-1}(3,65/1,86)$
$\theta = 27,00$

ACTIVITIES/ ASSESSMENT

1. Draw the following forces as vectors on the Cartesian plane originating at the origin:
- $F_1$ = 3,7 N in the positive x-direction
- $F_2$ = 4,9 N in the positive *y*-direction

2. Draw the following forces as vectors on the Cartesian plane:
- $F_1$ = 4,3 N in the positive x-direction
- $F_2$ = 1,7 N in the negative x-direction
- $F_3$ = 8,3 N in the positive *y*-direction

3. Find the resultant in the x-direction, $R_x$, and y-direction, $R_y$ for the following forces:
- $F_1$ = 4,8 N in the positive x-direction
- $F_2$ = 3,2 N in the negative x-direction
- $F_3$ = 1,9 N in the positive *y*-direction
- $F_4$ = 2,1 N in the negative *y*-direction

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| CONSOLIDATION | **SUMMARY**<br>• A vector has a magnitude and direction.<br>• Vectors can be used to represent many physical quantities that have a magnitude and direction, like forces.<br>• Vectors may be represented as arrows where the length of the arrow indicates the magnitude and the arrowhead indicates the direction of the vector.<br>• Vectors in two dimensions can be drawn on the Cartesian plane. |
| VALUES | Accurate graphic representation is important as a language of communication.<br>Graphic diagrams contain a subject language one can read. |