SUBJECT and GRADE	Mathematics Grade 10
TERM 1	Week 6: Exponents, Equations and Inequalities
TOPIC	Literal and simultaneous equations and word problems
AIMS OF LESSON	To solve different equations which are in the form of letters, simultaneous and words.
RESOURCES	***Paper based resources*** *Please refer to the chapter in your textbook on Solving linear equations* ***Digital resources*** *https://www.youtube.com/watch?v=gqSfw2gmMsg ; https://www.youtube.com/watch?v=UbYnNAyShVM* *https://www.youtube.com/watch?v=IMPaJYdccyQ ; https://www.youtube.com/watch?v=Lwto-lQzmec* *https://www.youtube.com/watch?v=xKH1Evwu150 ; https://www.youtube.com/watch?v=hfOkn-se7cQ*
INTRODUCTION	In this week's lessons we will focus on ways to solve Literal equations, solve systems of equations and look at word problems that lead to solving linear equations. All these topics are not only very important for further study in Mathematics, but also in other subjects like the Sciences, Economics and Geography
CONCEPTS/ SKILLS	7. Literal equations 8. Linear equations 9. Changing words to equations
Lesson 1	**Literal equations:** Changing the subject of a formula

Literal equations: equations involving different letters, e.g. $y = 2x + 3$; $A = xr^2$, etc.
To change the subject of the equation (to solve a specific letter) follow the steps as in solving equations as illustrated below:

Solve for x:

Example 1. $ax = c - bx$

$\therefore ax + bx = c$

$\therefore x(a + b) = c$

$\therefore x = \dfrac{c}{a + b}$

- Isolate the unknown on the LHS by doing inverse operations
- Factorise (usually CF)
- Solve by dividing by coeff. of unknown

Example 2: $\frac{x}{a} - \frac{x}{b} = b - a$

$x\left(\dfrac{1}{a} - \dfrac{1}{b}\right) = b - a$

$x\left(\dfrac{b - a}{ab}\right) = b - a$

$x = (b - a) \times \dfrac{\boldsymbol{ab}}{\boldsymbol{(b - a)}}$

$= ab$

1. Take out CF of x
2. Simplify the LHS by writing fractions with a common denominator : LCM of denominators
3. Solve by dividing by coeff. of x (× reciprocal of coeff.)

Example 3: The formula to convert temperature from °Celsius to °Fahrenheit is given by: $\mathrm{F} = \frac{9}{5}\mathrm{C} + 32$

a) Make C the subject of the formula

b) Determine the temperature in °C if it is 68 °F

Solution: a) $\frac{9}{5}\mathrm{C} + 32 = \mathrm{F}$

$\therefore \frac{9}{5}\mathrm{C} + 32 - \mathbf{32} = \mathrm{F} - \mathbf{32}$

$\therefore \frac{9}{5}\mathrm{C} = \mathrm{F} - 32$

$\therefore \dfrac{\frac{9}{5}\mathrm{C}}{\frac{9}{5}} = \dfrac{\mathrm{F} - 32}{\frac{9}{5}}$

$\therefore \mathrm{C} = \frac{5}{9}(\mathrm{F} - 32)$

1. Write the side with C on the LHS (you can leave it on RHS and work from there)
2. Isolate C by subtracting 32 both sides
3. Solve C by dividing by $\frac{9}{5}$ $\left(\times \frac{5}{9}\right)$

b) Substitute $\mathrm{F} = 68$ in equation: $\mathrm{C} = \frac{5}{9}(68 - 32)$

$= 20\ °\mathrm{C}$

Example 4: If $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$, make f the subject of the formula. (Solve for f)

Solution: $\frac{1}{u}+\frac{1}{v}=\frac{1}{f}$

$\therefore \frac{v+u}{uv}=\frac{1}{f}$

$\therefore \frac{uv}{v+u}=f$

1. Since f is alone on RHS, simplify LHS
2. LCM of denominators on LHS is uv
3. Write LHS with LCM as common denominator (do not × with LCM since we have f alone on RHS)
4. To get f we take reciprocals both sides

Example 5: The area of a flat ring is given by: $\mathrm{A}=\pi(\mathrm{R}^2-r^2)$.

a) Make r the subject of the formula

b) Calculate the value of r if $\mathrm{A}=2{,}86\ \mathrm{cm}^2$; $\pi=\frac{22}{7}$ and $\mathrm{R}=1$ cm

Solution: $\pi(\mathrm{R}^2-r^2)=\mathrm{A}$

$\therefore \frac{\pi(\mathrm{R}^2-r^2)}{\pi}=\frac{\mathrm{A}}{\pi} \Rightarrow \mathrm{R}^2-r^2=\frac{\mathrm{A}}{\pi}$

$\therefore \mathrm{R}^2-r^2-\mathbf{R}^2=\frac{\mathrm{A}}{\pi}-\mathbf{R}^2$

$\therefore -r^2=\frac{\mathrm{A}}{\pi}-\mathrm{R}^2$

$\therefore -(-r^2)=-\left(\frac{\mathrm{A}}{\pi}-\mathrm{R}^2\right)$

$\therefore r^2=\mathrm{R}^2-\frac{\mathrm{A}}{\pi}$

$\therefore r=\sqrt{\mathrm{R}^2-\frac{\mathrm{A}}{\pi}}$

7. Write term(s) with r on LHS
8. We want to isolate $r \Rightarrow$Divide by π
9. Subtract R^2 both sides
10. Divide by -1
11. To remove square, take **square root** both sides $\pm\sqrt{r^2}=r$ (we use only + since we working with quantities)

b) $r=\sqrt{1^2-\frac{2{,}86}{\frac{22}{7}}}$

$\therefore r=\sqrt{1-\left(2{,}86\times\frac{7}{22}\right)}$

$=0{,}3$ cm

Substitute given values in formula for r and use calculator for answer.

CAN YOU?

A. Solve for x:
1. $ax=a^2-b^2-bx$
2. $\frac{x}{a}-b=c$
3. $\frac{x}{m}+\frac{x}{n}=\frac{m+n}{mn}$
4. $p=m-\frac{x^2}{2}$

A. Answers:
1. $a-b$
2. $ac+ab$
3. 1
4. $\pm\sqrt{2(m-p)}$

C. Answers:
1. $a=\frac{2S}{n}-l$
2. $a=3$

D. $r=3{,}27$

E.
1. $r=100\left(\sqrt[n]{\frac{A}{P}}-1\right)$
2. $r=10{,}07$

B. Change the subject of the following as indicated:
1. $E=mc^2$ $\quad m=$
2. $\mathrm{F}=\frac{mv^2}{gr}$ $\quad g=$
3. $\mathrm{V}=\pi r^2h$ $\quad r=$
4. $e=\frac{\mathrm{E}}{\mathrm{R}+r}$ $\quad \mathrm{R}=$
5. $v=m\sqrt{a^2-b}$ $\quad a=$
6. $\mathrm{S}=\frac{n}{2}[2a+(n-1)d]$ $\quad d=$
7. $\mathrm{T}_n=ar^n$ $\quad r=$

B. Answers:
1. $m=\frac{E}{c^2}$
2. $g=\frac{mv^2}{Fr}$
3. $r=\sqrt{\frac{V}{\pi h}}$
4. $R=\frac{E}{e}-r$
5. $a=\sqrt{\left(\frac{v}{m}\right)^2+b}$
6. $d=\frac{\frac{2S}{n}-2a}{n-1}$
7. $r=\sqrt[n]{\frac{T_n}{a}}$

C. 1. Derive a formula for a from $\mathrm{S}=\frac{n}{2}(a+l)$

2. Find a if $\mathrm{S}=860$; $n=20$ and $l=83$

D. If $\mathrm{V}=\frac{4}{3}\pi r^3$, determine r if $\mathrm{V}=147$, correct to 2 decimals.

E. Given $\mathrm{A}=P\left(1+\frac{r}{100}\right)^n$
1. Make r the subject of the formula
2. Find r if $\mathrm{A}=2200, \mathrm{P}=200$ and $n=25$, to 2 decimals.

Lesson 2 + 3	Simultaneous equations

If we look at $x + y = 2$, we can solve y for any x −value (there are infinitely many solutions), e.g.:

x	-1	0	1	2	$\mathbf{3}$
y if $x + y = 2$ $(y = 2 - x)$	3	2	1	0	$\mathbf{-1}$

We say that each ordered pair $(x;\ y)$ like $(2;\ 0)$ **satisfy** the equation (if we substitute the x −value, we get the y −value)

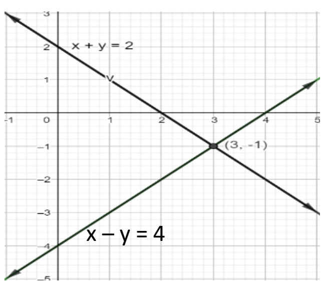
In the same way if we have $x - y = 4$:

x	-1	0	1	2	$\mathbf{3}$
y if $x - y = 4$ $(y = x - 4)$	-5	-4	-3	-2	$\mathbf{-1}$

Looking at both equations however, we see **only 1** ordered pair $(\mathbf{3};\ \mathbf{-1})$ that satisfy BOTH equations. This solution is **unique** and we say: $(3;\ -1)$ or $x = 3$ and $y = -1$ are the simultaneous solutions of $x + y = 2$ and $x - y = 4$

We can solve a system of simultaneous equations graphically and algebraically

Graphically: if we draw the graphs representing the 2 equations (straight lines), we have:



We can clearly see that $(3;\ -1)$ is **the point of intersection** of the 2 lines (we will explore this method more when we do Functions)

There are 2 methods to solve simultaneous equations **Algebraically**:

We will use the 2 given equations to demonstrate the different methods.

Method of Substitution

$x + y = 2 \quad \ldots (1)$

$x - y = 4 \quad \ldots (2)$

From eqn (2):

$x = y + 4 \quad \ldots (3)$

Subst. eqn (3) into eqn **(1)**

$\therefore (\mathbf{y + 4}) + y = 2$

$\therefore 2y + 4 = 2$

$\therefore 2y = -2$

$\therefore \mathbf{y = -1}$

Subst. $y = -1$ into eqn (3): $x = y + 4 \Rightarrow x = (-1) + 4$

$\therefore \mathbf{x = 3}$

- Number the equations (1) and (2)
- Take 1 eqn and write x or y in terms of the other – number it (3) – usually the 1 with coefficient ± 1 (why?)
- **Substitute** eqn (3) into eqn **not used**
- Solve the variable
- Substitute this value back into eqn (3) to solve other variable

Method of Elimination

$x + y = 2 \quad \ldots(1)$

$x - y = 4 \quad \ldots(2)$

(1) + (2): $2x = 6$ (Since the coeff. of y are different)

$\therefore x = 3$

Subst. $x = 3$ in (1)

$\therefore (3) + y = 2$

$\therefore y = 2 - 3$

$\therefore y = -1$

(if we subst. $\boldsymbol{x} = \mathbf{3}$ in (2):

$\therefore (3) - y = 4$

$\therefore -y = 4 - 3 = 1$

$\therefore \boldsymbol{y} = -\mathbf{1}$

1. Number the equations (1) and (2)
2. Eliminate 1 of the variables (if the **coefficients** are **the same**: **sub**tract the 2 equations; if the coefficients are **different**, **add** the 2 equations.
3. Solve the variable
4. Substitute this value back into eqn (1) or (2) to solve other variable

In the following examples, we will explore both methods with the same set of equations.

Elimination:

Solve for x and y simultaneously:

Example 1: $x + 2y = 5$ and $x - y = \frac{1}{2}$

Solution: $x + 2y = 5 \quad \ldots(1)$

$- \; x \; -\!\!+ \; y = -\frac{1}{2} \quad \ldots(2)$

(1) − (2): $3y = 4\frac{1}{2} = \frac{9}{2}$ (since the coeff. of x are the same)

$\therefore y = \frac{9}{2} \times \frac{1}{3}$

Is we ÷ 3, it is the same as $\times \frac{1}{3}$

$\therefore y = \frac{3}{2}$

Subst. $\therefore y = \frac{3}{2}$ in (1) $\therefore x + 2\left(\frac{3}{2}\right) = 5$

$\therefore x + 3 = 5$

$\therefore x = 2$

Substitution:

Solve for x and y simultaneously:

Example 1: $x + 2y = 5$ and $x - y = \frac{1}{2}$

Solution: $x + 2y = 5 \quad \ldots(1)$

$x - y = \frac{1}{2} \quad \ldots(2)$

From (1): $x = 5 - 2y \quad \ldots(3)$

Subst. (3) into (2) $\therefore (5 - 2y) - y = \frac{1}{2}$

$\therefore 5 - 3y = \frac{1}{2}$

$\therefore -3y = \frac{1}{2} - 5 = -4\frac{1}{2} = -\frac{9}{2}$

$\therefore y = -\frac{9}{2} \times -\frac{1}{3} = \frac{3}{2}$

Subst. $y = \frac{3}{2}$ in (3) $\therefore x = 5 - 2\left(\frac{3}{2}\right)$

$\therefore x = 2$

Can be subst. into any of the 3 equations, but into (3) is easier – why?

Example 2: $2x - 3y = 5$ and $6y + 3x = 11$

Solution: $2x - 3y = 5$ …(1)

$\mathbf{3x} + \mathbf{6y} = 11$ …(2)

Write like terms underneath each other

(1) $\times 2$: $4x - 6y = 10$ …(3)

(2) +(3): $7x \quad = 21$

Since none of the coeff. are the same, we can make the coeff. of x the same by $\times$ each term of (1) by 2: Number it (3)

$\therefore x = 3$

Subst. $x = 3$ into (1): $\therefore 2(3) - 3y = 5$

$\therefore 6 - 3y = 5$

$\therefore -3y = -1$

$\therefore y = \frac{1}{3}$

Example 2: $2x - 3y = 5$ and $6y + 3x = 11$

Solution: $2x - 3y = 5$ …(1)

$\mathbf{3x} + \mathbf{6y} = 11$ …(2)

Write like terms underneath each other

From (1): $2x = 3y + 5 \quad \Rightarrow x = \frac{3y+5}{2}$ …(3)

Subst. (3) into (2): $\therefore 3\left(\frac{3y+5}{2}\right) + 6y = 11$

$\times$ LCM: 2 $\quad \therefore 3(3y + 5) + 12y = 22$

$\therefore 9y + 15 + 12y = 22$

$\therefore 21y = 7 \quad \Rightarrow y = \frac{1}{3}$

Subst. $y = \frac{1}{3}$ into (3) $\quad x = \frac{3\left(\frac{1}{3}\right)+5}{2} \quad \Rightarrow x = 2$

From the 2 examples it should be clear that we use:

- Substitution when the coefficient of 1 of the variables in any of the 2 equations is ± 1 to avoid working with FRACTIONS or where you can get 1 coefficient ± 1 by simplifying an equation, e.g. ($2x - 8y = 6 \Rightarrow \div 2$: $\boldsymbol{x} - 4y = 3$)
- Elimination otherwise, unless you LIKE working with fractions

Example 3: Solve for x and y simultaneously if: $y = 2x - 1$ and $y = -3x + 4$

Solution: In this case we can just equate the 2 parts that's $=$ y and solve:

$\therefore 2x - 1 = -3x + 4$

$\therefore 5x = 5 \quad \Rightarrow x = 1$

Subst. $x = 1$ into any eqn: $\therefore y = 2(1) - 1$

$\therefore y = 1$

CAN YOU?

A. Use Substitution to solve x and y:

1. $x + 2y = 5$ and $4x + 2y = -10$
2. $2x - 4y = 10$ and $3x + 5y = -29$
3. $2x - 3y = 4$ and $4x - 3y = 20$

B. Use Elimination to solve x and y:

1. $x + 2y = -4$ and $3x - 2y = -4$
2. $2x + 4y = 7$ and $8x - 2y = 1$
3. $3x + 2y = 27$ and $3y + 2x = 28$

C. Solve the following systems of simultaneous equations (any method):

1. $3x - y = 1$ and $x + 2y = 5$
2. $-3x + 4y = 18$ and $4x + y = -5$
3. $y = 2x + 3$ and $y = -3x - 7$
4. $3x - 2y = 0$ and $3x - 4y = -1$
5. $5x - 2y = 6$ and $3x + 4y = 14$
6. $\frac{x+3}{2} + \frac{y-2}{4} = \frac{11}{4}$ and $\frac{2x-1}{3} + \frac{y+3}{2} = 4$
7. $y = -4x + 12$ and $y = 4x^2 - 8x - 3$ *
8. $\frac{6}{x} - \frac{1}{y} = 4$ and $\frac{9}{x} + 1 = \frac{-2}{y}$ **

A. Answers:

1. $x = -5; y = 5$
2. $x = -3; y = -4$
3. $x = 8; y = 4$

B 1. $x = -2; y = -1$
2. $x = \frac{1}{2}; y = \frac{3}{2}$
3. $x = 5; y = 6$

C. Answers:

1. $x = 1; y = 2$
2. $x = -2; y = 3$
3. $x = -2; y = -1$
4. $x = \frac{1}{3}; y = \frac{1}{2}$
5. $x = 2; y = 2$
6. $x = 2; y = 3$
7. $x = -\frac{3}{2}; y = 18$
Or $x = \frac{5}{2}; y = -9$
8. $x = 3; y = -\frac{1}{2}$

Lesson 4 + 5	Word Problems (Mathematical modelling)

To solve word problems:

- READ through the problem to make sure you understand what is required.
- Let x be one of the unknowns (usually what was asked) – if more than 1, make the smallest x
- Now write the other unknown in terms of x (you can also make it y and work with simultaneous equations)
- Set up an equation from the remaining information and solve for x (and y)
- Check the validity of your answer(s)

Key words:
Addition: sum, added to, increased by, more than, together, etc.
Subtraction: difference, subtracted, decreased by, less than, take away, etc.
Multiplication: multiplied by, times, twice, trice, double, product, etc.
Division: divided by, quotient, share among, etc.
Equals: is, give, was, equal, in total, becomes, etc.

Give mathematical expressions for the following descriptions:
E.g. - My age 10 years from now: $x + 10$; my age 5 years ago: $x - 5$
- 4 times my age: $4x$
- 4 times my age 3 years from now: $4(x + 3)$
- Difference: highest – lowest; oldest – youngest; **slowest - fastest**
- 2 consecutive numbers: x and $x + 1$

Remember:

- Distance = speed × time
- Total price = number of items × price for 1 item
- Profit = selling price – purchace price
- Area of rectangle = $l \times b$; Perimeter of rectangle = $2(l + b)$

Example 1: The sum of 2 consecutive numbers is 71. What are the numbers?

Solution: Let x be one number ⇒ other number = $x + 1$
$\therefore x + (x + 1) = 71$
$\therefore 2x = 70 \quad \Rightarrow \quad x = 35$ and $x + 1 = 35 + 1 = 36$
∴The numbers are 35 and 36

Example 2: Take a certain number and add 5 to it. Divide the result by 3 and then subtract 1. This gives an answer of 2. What is the number?

Solution: Let the number = x
$\therefore \frac{x+5}{3} - 1 = 2$
× LCM: 3 $\quad \therefore x + 5 - 3(1) = 3(2)$
$\therefore x + 2 = 6$
$\therefore x = 4$

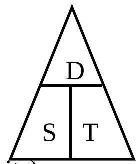
Example 3: A father and his son are 36 years old altogether. 7 years from now, the father will be 4 times the age of his son then. What are there ages now?

Solution: Let son = x years ⇒ father = $36 - x$ (There sum is 36)
7 years from now: son = $x + 7$ and **father** = $36 - x + 7 = 43 - x$
But **father** also $4(x + 7)$
$\therefore 4(x + 7) = 43 - x$
$\therefore 4x + 28 = 43 - x$
$\therefore 5x = 15$
$\therefore x = 3$ ∴son is 3 years old and father is $36 - 3 = 33$ years

Example 4: I have twice as many 20c pieces than 10c pieces. If I have R4,50 in total, how many 10c and 20c pieces do I have?

Solution: Let 10c pieces = x ⇒ 20c pieces = $2x$
Total Amount: $10(x) + 20(2x) = 450$ (Price)
$\therefore 10x + 40x = 450$
$\therefore 50x = 450$
$\therefore x = 9$
∴ there are **9** 10c pieces and 2(9) = **18** 20c pieces

Example 5: Abel rides his bike for 8 km at 30 km/h. Then his bike breaks down and he runs at 10 km/h to his destination. If the total distance took him 28 minutes, how far did he run?



Solution: Distance = speed × time

For distance/speed/time problems, it is best to draw a table:
Let the distance ran = $x\ km$. Now fill in the other values (Note the units)

	Rides	Runs
D (km)	8	x
S (km/h)	30	10
T (h)	$\frac{8}{30}$	$\frac{x}{10}$

Time = $\frac{distance}{speed}$

Total time: $\frac{8}{30} + \frac{x}{10} = \frac{28}{60}$

28 minutes = $\frac{28}{60}$ hours

×LCM: 60: $\therefore 2(8) + 6(x) = 28$
$\therefore 16 + 6x = 28$
$\therefore 6x = 12$
$\therefore x = 2$
$\therefore$ Abel ran for 2 km

Example 6: The sum of the digits in a 2 digit number is 9. If the digits are swapped, the new number is 9 less than the original number. What is the original number.

Solution: Let the digits be x(tens digit) and y(units digit)
$\Rightarrow$ number: $10x + y$
Now: $x + y = 9$ …(1)
If swapped: new number: $10y + x$
Old number – new number = 9 $\therefore 10x + y - (10y + x) = 9$
$\therefore 10x + y - 10y - x = 9$
$\therefore 9x - 9y = 9 \Rightarrow x - y = 1$ …(2)
$(1) + (2)$: $2x = 10 \Rightarrow x = 5$ and $y = 4$
$\therefore$ the original number is $10(5) + 4 = 54$

CAN YOU?
Solve the following word problems:

1. The sum of 3 consecutive even numbers is 84. What are the numbers?
2. Take a number and subtract 7 from it. If you multiply that by 3 and then subtract 5, the answer will be 10. What is the number?
3. Sipho and Jenny are 34 years old altogether. 25 years from now, Sipho will be 4 times the age of what Jenny was 9 years ago. What is Jenny's age?
4. A father is 9 times the age of his son. 3 years from now the father will be only 5 times as old as his son then. What is the age of the father now?
5. Petro has 15 coins consisting of 10c and 5c pieces. If the total value of the coins is R1,15, how many 10c pieces does she have?
6. A farmer bought sheep and cattle for an amount of R53 800. The price of sheep was R250 a piece and the cattle were R730 each. If he bought 100 animals, how many of each did he buy?
7. A rectangle is 5 m longer than is is wide. If the perimeter of the rectangle is 30 m, calculate its length and breadth.
8. Siya walked to the hall at 4 km/h and ran back home at 9 km/h. If it took him 4 hours and 20 minutes to get to the hall and back, how far was he from the hall?
9. Two runners set off at 8h00 and run in oppsite directions. Ann runs at 12 km/h and Sandra at 8 km/h. At what time will they be 90 km apart?
10. An earoplane takes 4 h less time to travel a distance of 240 km than a car that travels at a fifth the speed of the earoplane. Calculate the speed of the car.
11. The sum of the units digit and tens digit of a 2 digit number is 8. If the digits are reversed, the new number will be 18 more than the original number. What is the new number?

Answers:
1. 26, 28 and 30
2. 12
3. Jenny=19; Sipho=15
4. Son=3; father=27
5. 5c = 7; 10c=8
6. 40 sheep; 60 cattle

Answers:
7. Length = 10 and breadth =5
8. 12 km
9. 12:30
10. Speed of car=48 km/ h
11. 53

ACTIVITIES	*Consider other exercises from your Mathematics Textbook*
VALUES	*Dear learner. Success isn't something that just happens - success is **learned**, success is **practiced** and then it is **shared - Sparky Anderson**. Keep learning and practicing Mathematics every day and you'll will reap SUCCESS!*