SUBJECT and GRADE	Mathematics Grade 10	
TERM 1	Week 5: Exponents, Equations and Inequalities	
TOPIC	Linear and quadratic equations and Inequalities	
AIMS OF LESSON	To solve different linear and quadratic equations	
RESOURCES	***Paper based resources***	***Digital resources***
	Please refer to the chapter in your textbook on Solving linear equations	https://www.youtube.com/watch?v=TkL0Iqs9mpY https://www.youtube.com/watch?v=GmMX3-nTWbE https://www.youtube.com/watch?v=WOn7c7PdnTk https://www.youtube.com/watch?v=wwDfD4iGBDE https://www.youtube.com/watch?v=fOnLQ_5mQic https://www.youtube.com/watch?v=AuYCyMCmovU
INTRODUCTION	In this week's lessons we will focus on ways to solve Linear and quadratic Equations.	
CONCEPTS/ SKILLS	1. Number systems 2. LCM of denominator 3. Simplifying/ Factorising expressions 4. Solve equations	
Lesson 1	Revision of Gr 9: Solving Linear Equations	

Expression: $2x^3 - \frac{4x}{\sqrt{x}} + 3$

Expressions can only be **simplified**

Equation: $2x^3 - \frac{4x}{\sqrt{x}} + 3 = 2x - 3$

Equation: when we **equate** two expressions; LHS = RHS; Equations can be **solved**

Linear equations: highest exponent of the unknown (usually x) is 1
Quadratic equations: highest exponent is 2
Cubic equations: highest exponent is 3

Solve for x:

Example 1: $2x + 3 = 7$

Isolate terms with x on LHS by adding -3 both sides

$\therefore 2x + 3 \mathbf{- 3} = 7 \mathbf{- 3}$

$\therefore 2x = 4$

Solve for x by dividing both sides by 2

$\therefore \frac{2x}{2} = \frac{4}{2}$

$\therefore x = 2$

Example 2:

$2x + 3 = 2 - 3(x + 3)$

$\therefore 2x + 3 = 2 - \mathbf{3x - 9}$

$\therefore 2x + 3 - \mathbf{3} + \mathbf{3x} = 2 - 3x - 9 - \mathbf{3} + \mathbf{3x}$

Isolate the unknowns on LHS: remove $+3$ on the LHS and $-3x$ on the RHS by doing **inverse operations** both sides of $=$ sign

$\therefore 2x + 3x = 2 - 9 - 3$

$\therefore 5x = -10$

$+$ and $-$ are **Inverse operations** while $\times$ and $\div$ are inverse operations of each other

$\therefore \frac{5x}{5} = \frac{-10}{5}$

$\therefore x = -2$

You can TEST the correctness of your answers by substituting the value of x into the RHS and LHS of the original equation to see if they are equal

Example 3: $\frac{x+2}{3} - \frac{x-1}{2} = 4$ LCM of 2 and 3 is **6**

Multiply **each term** by 6:

$$\therefore \frac{\mathbf{6}(x+2)}{3} - \frac{\mathbf{6}(x-1)}{2} = (\mathbf{6})4$$

$$\therefore 2(x+2) - 3(x-1) = 24$$
$$\therefore 2x + 4 - 3x + 3 = 24$$
$$\therefore 2x - 3x = 24 - 3 - 4$$
$$\therefore -x = 17$$
$$\therefore x = -17$$

- **Factorise** denominators, if any
- Remove fractions by **multiplying** each term with the **LCM** of the denominators
- Remove brackets (**distributive law**)
- Isolate the unknown on LHS by doing the **inverse operation** both sides of = sign
- Add like terms
- Solve for x by dividing with the coefficient of x both sides

NOTE:

- if we solve: $4(x-2) = 4x - 8$

  $\therefore 4x - 8 = 4x - 8$ this will be TRUE for **any** $\mathbb{R}$ **values** of x

  Answer: $x \epsilon\ \mathbb{R}$

  - This type of equation we call an **IDENTITY**
- If we solve: $4(x-2) = 4x$

  $\therefore\ 4x - 8 = 4x$

  $\therefore -8 = 0$ which can never be true

  Answer: **No** $\mathbb{R}$ **solution** for x

- If we have one term on LHS and one term on RHS we can also cross-multiply like below:

$$\frac{5x}{3} = \frac{x+1}{2}$$

$$\therefore 2(5x) = 3(x+1)$$
$$\therefore 10x = 3x + 3$$
$$\therefore 7x = 3$$
$$\therefore x = \frac{3}{7}$$

CAN YOU?

Solve the following equations:

1. $2x - 5 = 3$
2. $4x + 6 = x - 3$
3. $5a - 3(a+1) = 2 - 3a$
4. $2(x+2) = 6(x+1) - (x-4)$
5. $(2b-5)(3b+2) = 2(3b^2 - 4b + 1)$
6. $2(x+1) - (3-2x) = 2x - (7-2x)$
7. $\frac{x-2}{3} = \frac{x-3}{4}$
8. $\frac{5x+8}{6} - \frac{x}{4} = \frac{2x-9}{3}$
9. $\frac{3(y-1)}{2} = y - 2 + \frac{y+1}{2}$
10. $\frac{3}{2}(x+3) - \frac{2x}{3} = 2 - \frac{4x-3}{6}$

Answers:

1. $x = 4$
2. $x = -3$
3. $a = 1$
4. $x = -2$
5. $b = -4$
6. No $\mathbb{R}$ solution
7. $x = -1$
8. $x = 52$
9. $y \epsilon\ \mathbb{R}$
10. $x = -\frac{4}{3}$

Lesson 2	**Linear equations: Rational expressions (fractions with unknown in denominator)**

NOTE: When working with Rational equations: **denominators $\neq$ 0** since division by 0 is undefined. Hence, we have **restrictions** in the value of the unknown (x)

x cannot have these values, otherwise we are dividing by 0

Solve the following equations:

Example 1: $\frac{5}{x+2} + \frac{3}{2-x} = \frac{2}{x}$ LCM: $x(x+2)(2-x)$; $x \neq 0;\ -2;\ +2$

$\times$ each term with LCM:

You don't need to show this step, but for practice's sake, we'll show this now

$$\therefore \frac{5}{x+2} \times x(x+2)(2-x) + \frac{3}{2-x} \times x(x+2)(2-x) = \frac{2}{x} \times x(x+2)(2-x)$$

$$\therefore 5x(2-x) + 3x(x+2) = 2(x+2)(2-x)$$
$$\therefore 10x - 5x^2 + 3x^2 + 6x = 2(4-x^2)$$
$$\therefore 10x - 5x^2 + 3x^2 + 6x = 8 - 2x^2$$
$$\therefore 16x = 8$$
$$\therefore x = \frac{1}{2}$$

Note: $(x+2) = (2+x)$ leading to difference of two squares

Example 2: $\frac{2x-5}{x^2-2x-8} = \frac{1}{2x-8}$ apply method

$$\therefore \frac{2x-5}{(x+2)(x-4)} = \frac{1}{2(x-4)}$$ LCM: $2(x+2)(x-4)$; $x \neq -2;\ \mathbf{4}$

$\times$ each term with LCM:

$$\therefore 2(2x-5) = x+2$$
$$\therefore 4x - 10 = x + 2$$
$$\therefore 3x = 12$$
$\therefore x = \mathbf{4}$, but this is a restriction!!!

$\therefore$ Answer: True for no values of x

For each "NEW" term: LCM $\div$ Denominator $\times$ numerator

A Method to solve linear equations:

- **Factorise** denominators, if any
- Remove fractions by **multiplying** each term with the **LCM** of the denominators
- Remove brackets (**distributive law**)
- Isolate the unknown on LHS by doing the **inverse operation** both sides of = sign
- Add like terms
- Solve for x by dividing with the coefficient of x both sides

CAN YOU?

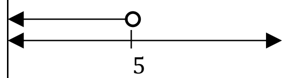
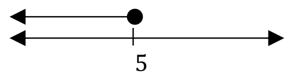
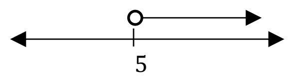
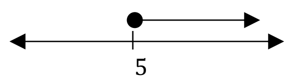
Solve the following equations:

1. $\frac{x}{2} - \frac{x}{3} = 2$
2. $\frac{x+2}{4} - \frac{x-6}{3} = \frac{1}{2}$
3. $\frac{2}{x} = \frac{3}{x-2}$
4. $1 + \frac{3}{2x-1} = \frac{4}{3}$
5. $\frac{1}{m-1} = \frac{3}{m} - \frac{2}{m+2}$
6. $\frac{1}{x^2-x} + \frac{2}{x-1} = \frac{1}{x}$
7. $\frac{1}{x+2} + \frac{3}{x^2-4} = \frac{2}{2-x}$
8. $\frac{1}{2x^2-x-3} + \frac{2}{x^2-1} = \frac{3}{2x^2-5x+3}$

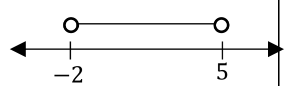
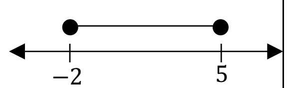
Answers:

1. $x = 12$
2. $x = 24$
3. $x = -4$
4. $x = 5$
5. $m = 2$
6. $x = -2$
7. $x = -1\frac{2}{3}$
8. $x = 5$

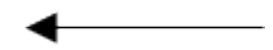
Lesson 3 + 4	Linear inequalities

Symbol	Meaning	Example	Number line
$<$	Less than	$x < 5$ Read: "x is less than 5 (all x values less than 5)"	
$\leq$	Less than or equal to	$x \leq 5$ "x is less than or equal to 5"	
$>$	Greater than	$x > 5$ "x is greater than 5"	
$\geq$	Greater than or equal to	$x \geq 5$ "x is greater than or equal to 5"	

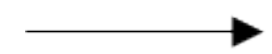
Representations of intervals:

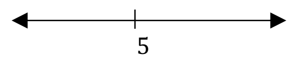
Interval Notation	Set-builder notation	Number line
$x \in (-2; 5)$ Read: All x values (Real numbers) **between** -2 and 5 *Open interval*	$\{x: -2 < x < 5, x \in \mathbb{R}\}$ Read: All x values, SUCH THAT x is greater than -2, but less than 5 and x is a Real number	
$x \in [-2; 5]$ All x values (Real numbers) from -2 to 5 (inclusive) *Closed interval*	$\{x: -2 \leq x \leq 5, x \in \mathbb{R}\}$ All x values, SUCH THAT x is greater than or equal to -2, but less than or equal to 5 and x is a Real number	

○ - shows that the number (5) is NOT INCLUDED in the interval

 shows all the $\mathbb{R}$ values to the left of 5

● - shows that the number (5) is INCLUDED in the interval

 shows all the $\mathbb{R}$ values to the right of 5

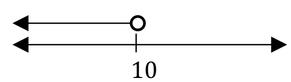
 Number line

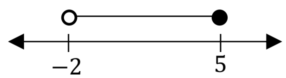
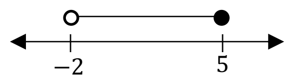
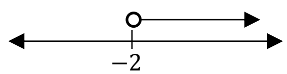
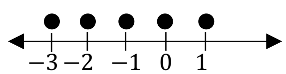
CAN YOU?

A. Show the following on a number line:

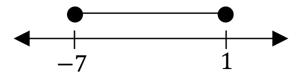
1. $x \in (7; 12)$
2. $\{x: 0 < x \leq 8, x \in \mathbb{R}\}$
3. $\{x: 5 \leq x < 10, x \in \mathbb{R}\}$
4. $x \in (-\infty; 3]$
5. $\{x: -1 \leq x < 3, x \in \mathbb{Z}\}$

B. Write in set-builder notation:

1. $x \in [0; 5)$
2. 
3. $x \in \{0; 1; 2; 3; 4\}$

$x \in [-2; 5)$ All real numbers from and including -2 up to, but not including/ excluding 5 *Closed-open or half open interval*	$\{x: -2 \leq x < 5, x \in \mathbb{R}\}$ All x values, SUCH THAT x is greater than or equal to -2, but (and) less than 5 and x is a Real number	
$x \in (-2; 5]$ All real numbers between -2 up to, and including 5 *Open-closed or half closed interval*	$\{x: -2 < x \leq 5, x \in \mathbb{R}\}$ All x values, SUCH THAT x is greater than -2, but less than or equal to 5 and x is a Real number	
$x \in (-2; \infty)$ All real numbers greater than -2 Note: ∞ (infinity) – always open	$\{x: x > -2, x \in \mathbb{R}\}$ All x values, SUCH THAT x is greater than -2 and x is a Real number	
NOTE: Interval notation is only used for **continuous intervals** (usually $x \in \mathbb{R}$) For sets of integers ($\mathbb{Z}$) we have, e.g.: $x \in \{-3;\ -2;\ -1; 0; 1\}$	$\{x: -4 < x < 2, x \in \mathbb{Z}\}$ Or $\{x: -3 \leq x \leq 1, x \in \mathbb{Z}\}$	Note: Only individual points 

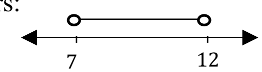
C. Write in Interval notation:

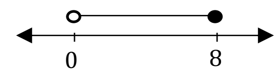
1. 

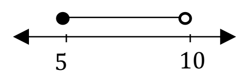
2. $\{x: x > -7, x \in \mathbb{R}\}$

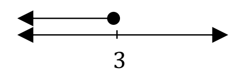
3. $\{x: -1 < x \leq 3, x \in \mathbb{Z}\}$

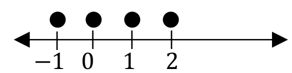
Answers:

A 1. 

2. 

3. 

4. 

5. 

B

1. $\{x: 0 \leq x < 5, x \in \mathbb{R}\}$
2. $\{x: x < 10, x \in \mathbb{R}\}$
3. $\{x: 0 \leq x \leq 4, x \in \mathbb{Z}\}$ OR $\{x: -1 < x < 5, x \in \mathbb{Z}\}$

C

1. $x \in [-7; 1]$
2. $x \in (-7; \infty)$
3. $x \in (-1; 3]$

Solving Linear equations:

Note: the following statement is true: $7 > 4$ (1)

If we add 6 both sides: $13 > 10$ is still true;

Also: $7 - 6 > 4 - 6 \Rightarrow 1 > -2$ is true

If we $\times$ both sides by 6 in (1): $42 > 24$ is still true;

Also: $\frac{7}{4} > \frac{4}{4} \Rightarrow 1\frac{3}{4} > 1$ is true.

However, if we $\times$ both sides by -1 in (1): $-7 > -4$ is NOT TRUE

The inequality sign must change!: $\mathbf{-7 < -4}$

Hence, if we $\times$ or $\div$ an inequality with a negative $(-)$ number, the inequality sign changes direction

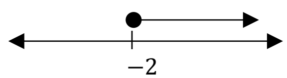
Solve the following linear inequalities and show the solution on a number line:

Example 1: $2 + 3x \geq x - 2$

$\therefore 3x - x \geq -2 - 2$

$\therefore 2x \geq -4$

$\therefore x \geq -2$



Example 2: $3(x - 1) - 2 < 6x + 4$

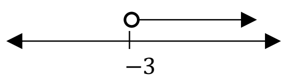
$\therefore 3x - 3 - 2 < 6x + 4$

$\therefore -3x < 9$

$\therefore \frac{-3x}{-3} > \frac{9}{-3}$

$\therefore x > -3$

$<$ changes to $>$ when we $\div (-3)$



Example 3: $-7 < 2x + 3 \leq 3$

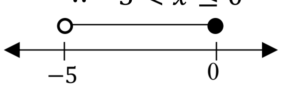
$\therefore -7 - \mathbf{3} < 2x + 3 - \mathbf{3} \leq 3 - \mathbf{3}$

$\therefore -10 < 2x \leq 0$

$\therefore -5 < x \leq 0$

Subtract 3 from all terms

Divide all the terms by 2



Example 4: $-3 < \frac{1-a}{3} < 4$

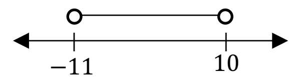
$\therefore -9 < 1 - a < 12$

$\therefore -10 < -a < 11$

$\therefore 10 > a > -11$

$\therefore -11 < a < 10$

- $\times 2$
- Subtract 1 from all terms
- $\div -1$: sign changes
- Rewrite solution; Smallest value on left



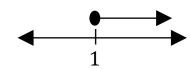
CAN YOU?

Solve the following linear inequalities and show the solution on a number line:

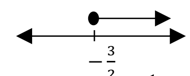
1. $9x - 7 \geq 5 - 3x$
2. $2(x - 3) \leq 4x - 3$
3. $5(a - 2) - 3a < 3 - (a - 2)$
4. $\frac{2x-5}{4} - \frac{5x-2}{3} > 0$
5. $-6 \leq 2x - 4 < 2$
6. $1 < \frac{3-x}{2} < 3$

Answers:

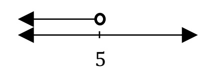
1. $x \geq 1$



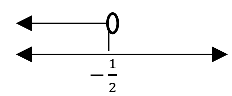
2. $x \geq -\frac{3}{2}$



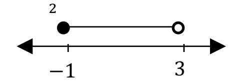
3. $a < 5$



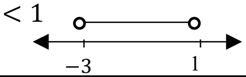
4. $x < -\frac{1}{2}$



5. $-1 \leq x < 3$



6. $-3 < x < 1$



Lesson 5	Quadratic equations: $ax^2 + bx + c = 0$

STANDARD FORM of quadratic equation: $\boldsymbol{ax^2 + bx + c = 0}$

NOTE: if $a.b = 0$
$\Rightarrow a = 0$ or $b = 0$

- **Factorise** denominators, if any
- **Remove fractions** by multiplying each term with the LCM of the denominators
- Remove brackets (distributive law)
- Write equation in **standard form**
- Factorise quadratic
- Apply Rule: if $a.b = 0$
  $\Rightarrow a = 0$ or $b = 0$
- The solutions of a quadratic equation are called the **ROOTS** of the quadratic (max of 2 roots)

Solve for x:

Example 1: $(x-2)(x+3) = 0$
$\therefore x - 2 = 0$ or $x + 3 = 0$
$\therefore x = 2$ or $x = -3$

Example 2: $x^2 - 5x - 6 = 0$
$\therefore (x-6)(x+1) = 0$
$\therefore x - 6 = 0$ or $x + 1 = 0$
$\therefore x = 6$ or $x = -1$

Example 3: $x^2 = -5x$
$\therefore x^2 + 5x = 0$
$\therefore x(x+5) = 0$
$\therefore x = 0$ or $x + 5 = 0$
$\therefore x = 0$ or $x = -5$

> **NOT:** $\div\, \boldsymbol{x}$: $\therefore \boldsymbol{x = -5}$
>
> **Never divide** by the unknown/ variable; you are (most probably) **dividing by zero**, which is not allowed, and you throw away one of the solutions: **Write in standard form!!**

Example 4: $(2x-1)(x+2) = 25$
$\therefore 2x^2 + 3x - 2 - 25 = 0$
$\therefore 2x^2 + 3x - 27 = 0$
$\therefore (2x+9)(x-3) = 0$
$\therefore 2x + 9 = 0$ or $x - 3 = 0$
$\therefore x = -\frac{9}{2}$ or $x = 3$

> NOT: $2x - 1 = 25$ **OR** $x + 2 = 25$
> Rule only applicable if $\boldsymbol{a.b = 0}$
> **Write in standard form!!**

Example 5: $4x^2 - 9 = 0$
$\therefore (2x-3)(2x+3) = 0$
$\therefore x = \frac{3}{2}$ or $x = -\frac{3}{2}$

> **Alternative:** $4x^2 = 9$
> $\therefore x^2 = \frac{9}{4}$
> $\therefore x = \pm\sqrt{\frac{9}{4}}$
> $\therefore x = \frac{3}{2}$ or $x = -\frac{3}{2}$

Example 6: $x^2 + 4 = 0$
$\therefore x^2 = -4$
$\therefore x = \pm\sqrt{-4}$

> x^2 is **ALWAYS** ≥ 0

> Non Real number

Answer: no $\mathbb{R}$ solution for x

Example 7: $\frac{x^2 - 2x - 3}{x - 3} = 2$ LCM $= (x-3)$; $x \neq 3$

$\times (x-3)$: $x^2 - 2x - 3 = 2(x-3)$
$\therefore x^2 - 2x - 3 = 2x - 6$
$\therefore x^2 - 4x + 3 = 0$
$\therefore (x+1)(x-3) = 0$
$\therefore x = -1$ or $x = 3$ Not applicable (not valid)
$\therefore x = -1$

CAN YOU?

Solve for x:

1. $(x+3)(x-1) = 0$
2. $-2(x+2)(x+3) = 0$
3. $5x(2x+1) = 0$
4. $x^2 + 14x + 48 = 0$
5. $2x^2 - 18 = 0$
6. $6x^2 = 5x + 6$
7. $(x-3)(x-2) = 12$
8. $\frac{x}{x-2} + \frac{2}{2-x} = \frac{1}{x-3}$
9. $\frac{3}{x} + \frac{3}{x^2 - x} = \frac{1}{x^2 - 1}$

Answers:

1. $x = -3$ or $x = 1$
2. $x = -2$ or $x = -3$
3. $x = 0$ or $x = -\frac{1}{2}$
4. $x = -6$ or $x = -8$
5. $x = 3$ or $x = -3$
6. $x = -\frac{2}{3}$ or $x = \frac{3}{2}$
7. $x = 6$ or $x = -1$
8. $x = 4$
9. $x = -\frac{2}{3}$

ACTIVITIES	*Consider other exercises from your Mathematics Textbook*
CONSOLIDATION	• Solving Linear equations: • **Factorise** denominators, if any • Remove fractions by **multiplying** each term with the **LCM** of the denominators • Remove brackets (**distributive law**) • Isolate the unknown on LHS by doing the **inverse operation** both sides of $=$ sign • Add like terms • Solve for x by dividing with the coefficient of x both sides • Linear inequalities: if we $\times$ or $\div$ an inequality with a negative $(-)$ number, the inequality sign changes direction • **STANDARD FORM** of quadratic equation: $\boldsymbol{ax^2 + bx + c = 0}$ • Solving quadratic equations: • **Factorise** denominators, if any • **Remove fractions** by multiplying each term with the LCM of the denominators • Remove brackets (distributive law) • Write equation in **standard form** • Factorise quadratic • Apply Rule: if $a.b = 0$ $\Rightarrow a = 0$ or $b = 0$ • The solutions of a quadratic equation are called the **ROOTS** of the quadratic (max of 2 roots)
VALUES	*Dear learner. Mathematics is not (just) about numbers, equations, computations, or algorithms: it is about understanding -* ***William Paul Thurston*** *Understanding comes through PRACTICE. Keep practicing Mathematics every day!*