SUBJECT and GRADE	Mathematics Grade 10
TERM 1	Week 4: Exponents, Equations and Inequalities
TOPIC	Number systems and Exponents
AIMS OF LESSON	To expose learners to the Real number system and introduce Exponents
RESOURCES	***Paper based resources***
	Please refer to the chapter in your textbook on Number Systems and then on Exponents
INTRODUCTION	In this lesson we will start to look at the Real number system in order to classify numbers and then move on to work with Exponents.
CONCEPTS/ SKILLS	1. Real numbers, Rational and Irrational numbers 2. Exponent laws and rules 3. Simplifying exponential expressions 4. Solving Exponential equations
Lesson 1 + 2	The Real number system

The set of Real Numbers, $\mathbb{R}$

The set is infinite and contains both Rational and Irrational Numbers.
Every Real Number can be represented as a point on the number line.

The Set of Rational Numbers, $\mathbb{Q}$

This is an infinite set of numbers that can be written in the form $\frac{a}{b}$, where both a and b are integers ($\mathbb{Z}$) and $b \neq 0$

- e.g.: $-2\frac{2}{3}$; $\frac{45}{32}$; $0{,}132$; -12; 1; $\sqrt{16}$; $\sqrt[3]{64}$; $2{,}1\dot{2}\dot{3}$; etc.

Note: $10 = \frac{10}{1}$ which is $\mathbb{Q}$
$0{,}745 = \frac{745}{1000}$ which is $\mathbb{Q}$
$0{,}\dot{1} = \frac{1}{9}$ which is $\mathbb{Q}$
$0 = \frac{0}{1}$ which is $\mathbb{Q}$
$\frac{a}{0}$ is **UNDEFINED.**
We **can NEVER** divide by 0.

$\mathbb{Q}$ is closed under addition, multiplication and subtraction (if you $\times, +,$ or $-$ $\mathbb{Q}$ numbers, your answer will be a $\mathbb{Q}$ number). This not true for division ($\frac{a}{0}$ is **UNDEFINED**)

- All Terminating decimals ($0{,}745$) and Recurring numbers ($0{,}\dot{1}$) are Rational numbers.
- **NOTE**: π is **not** $\mathbb{Q}$, although $\frac{22}{7}$ and $3{,}14$ are $\mathbb{Q}$ **approximated values** that we use.

The Set of Irrational Numbers, $\mathbb{Q}'$

This is an infinite set of numbers that cannot be written in the form $\frac{a}{b}$, where both a and b are integers

- E.g. $\sqrt{2}$; $\sqrt[3]{4}$; $\boldsymbol{\pi}$; $0{,}123\ldots$
- Square roots of numbers that are not perfect squares
- Cube roots of numbers that are not perfect cubes, etc.
- π

Example 1: Write $0{,}\dot{2}$ as a fraction.

$0{,}\dot{2} = 0{,}22222222\ldots$
Let $x = 0{,}2222222\ldots$ (1)
$\times 10$: $\therefore 10x = 2{,}22222222\ldots$ (2)
(2) − (1): $10x - x = 2{,}2222222\ldots - 0{,}22222222\ldots$
$\therefore 9x = 2$
$\therefore x = \frac{2}{9}$

Example 2: Show that $1{,}\dot{1}\dot{2}$ is a $\mathbb{Q}$ number.

$1{,}\dot{1}\dot{2} = 1{,}1212121212\ldots$
Let $x = 1{,}1212121212\ldots$ (1)
$\times 100$: $\therefore 100x = 112{,}12121212\ldots$ (2)
(2) − (1): $100x - x = 112{,}12121212\ldots - 1{,}12121212\ldots$
$\therefore 99x = 111$
$\therefore x = \frac{111}{99}$ which is $\mathbb{Q}$

Example 3: Between which two integers ($\mathbb{Z}$) do $\sqrt{21}$ lie?

Look for a **perfect square** that is just smaller than 21 and the one just bigger than 21, which is 16 and 25

$16 < 21 < 25$
$\therefore \sqrt{16} < \sqrt{21} < \sqrt{25}$
$\therefore 4 < \sqrt{21} < 5 \Rightarrow \sqrt{21}$ lie between 4 and 5

The Set of Integers, $\mathbb{Z}$

This infinite set contains a negative number, $-n$, to match every Natural Number, n.

- $\mathbb{Z} = \{....; -5; -4; -3; -2; -1; 0; 1; 2; 3; 4; 5; ....\}$
- This Set is closed under $+, -$ and $\times$

The Set of Whole Numbers (Counting numbers), $\mathbb{N}_0$

This is an infinite set of numbers.

- $\mathbb{N}_0 = \{0;1; 2; 3; 4; 5; ....\}$
- Closed under $+$ and $\times$. If $n \in \mathbb{N}_0$ then $n \times 0 = 0$ and $n + 0 = n$

The Set of Natural Numbers, $\mathbb{N}$

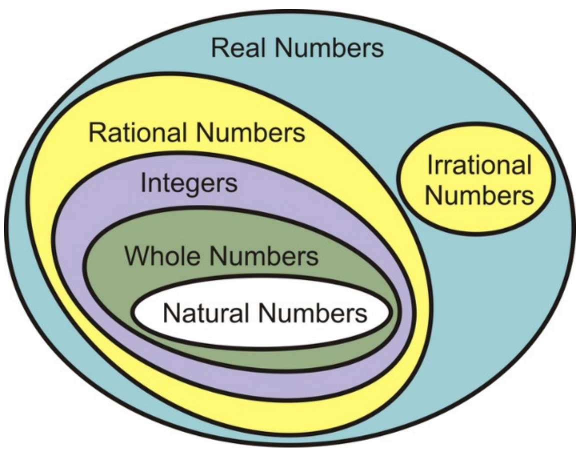
This is an infinite set of numbers.

- $\mathbb{N} = \{1; 2; 3; 4; 5; ....\}$
- This Set is closed under $+$ and $\times$.
- The set can be divided into Prime and Composite Numbers.
- Prime Numbers have only two factors 1 and itself. E.g. 2,3,5,7,11 and 13
- Composite Numbers have more than two factors

The Set of Non- Real Numbers, $\mathbb{R}'$

This is an infinite set of numbers.

- E.g. $\sqrt{-2}$; $\sqrt[6]{-20}$, etc.
- All even roots of negative numbers



To round off to n decimal places: Look at the digit to the right of this digit.

- If it is < 5 then the digit in the n −th place remains the same and the digits to the right falls away.
- If it is 5 or > 5 then the digit in the n −th place is increased by 1 and the digits to the right falls away.

Answers:

2. $\frac{19}{9}$
3. 8 and 9
4. a. 2,44
b. 0,0030
c. 43,701
d. 1,0
e. 24,9
f. 6040600

Example 4: Round the following off to the nearest decimal indicated:

1. 0,154 to 2 decimals $\Rightarrow$ 0,15
2. 2,376621 to 3 decimals $\Rightarrow$ 2,377
3. 5,764703 to 1 decimal $\Rightarrow$ 5,8

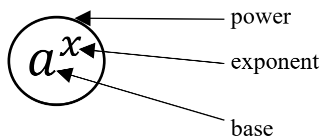
CAN YOU?

1. Classify the following numbers:

#	$\mathbb{R}$	$\mathbb{R}'$	$\mathbb{Q}$	$\mathbb{Q}'$	$\mathbb{Z}$	$\mathbb{N}_0$	$\mathbb{N}$	undef
e.g. $\sqrt{36}$	✓		✓		✓	✓	✓	
2π								
$\sqrt[3]{-36}$								
21,34 …								
$\sqrt{\frac{49}{4}}$								
$-\frac{7}{9}$								
$3{,}1\dot{7}8\dot{2}$								
7,85								
67								
$\sqrt{-16}$								
$\sqrt[3]{-8}$								
$\frac{25}{0}$								

2. Write $2,\dot{1}$ as a fraction
3. Between which two integers do $\sqrt{71}$ lie?
4. Round off to the nearest decimal indicated:
   (a) 2,43576 to 2 decimals
   (b) 0,00293019 to 4 decimals
   (c) 43,700951 to 3 decimals
   (d) 1,049 to 1 decimal
   (e) 24,876 to the nearest tenth
   (f) 6040599,87654 to the nearest whole number

Lesson 3	Exponents: Revision from Gr 9



Definition: $a^n = a \times a \times a \times a \times a \times a \times ...$ for n terms

$a^1 = a$

Exponential laws are only applicable if we have ONE TERM

Exponential laws: if $m, n \in \mathbb{Q}$:

Note: only **one** base in answer

1. $a^n \times a^m = a^{n+m}$ if we multiply 2 powers and their bases are the same, we add their exponents

Examples: $2^3 \times 2^4 = 2^{3+4} = 2^7$; $x^5.x^4 = x^9$; $a^{\frac{1}{2}} + a^{\frac{1}{2}} = a^1 = a$

Test: $8 \times 16 = 128 = 2^7$

2. $\frac{a^n}{a^m} = a^{n-m}$ if $n > m$ If we divide 2 powers with the same base, subtract the exponent of denominator from that of numerator

$= \frac{1}{a^{m-n}}$ if $m > n$

Examples: $\frac{3^4}{3^1} = 3^{4-1} = 3^3 = 27$; $\frac{a^6b^7}{a^7b^5} = \frac{b^{7-5}}{a^{7-6}} = \frac{b^2}{a}$

3. $(a^m)^n = a^{mn}$ E.g. $(2^3)^2 = 2^6 = 64$; $(x^3)^4 = x^{12}$; $(a^4)^{\frac{1}{2}} = a^{4\times\frac{1}{2}} = a^2$

4. $(ab)^n = a^n b^n$ E.g. $(3^2a^3b^{\frac{1}{2}})^2 = 3^4a^2b = 81a^2b$; $(2xy)^3 = 2^3x^3y^3 = 8x^3y^3$

5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ E.g. $\frac{2^3}{3^3} = \left(\frac{2}{3}\right)^3$; $\left(\frac{a^2}{b}\right)^4 = \frac{a^8}{b^4}$

Other deductions:

- $\frac{a^m}{a^m} = a^{m-m} = a^0 = 1$ E.g. $2^0 = 1$; $(x+y)^0 = 1$; $4a^0 = 4(1) = 4$
- $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$ E.g. $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; $\frac{1}{x^{-4}} = x^4$

NOTE:

- $-a^2 = -(a^2)$ and $(-a)^2 = (-a) \times (-a) = a^2 \Rightarrow -a^2 \neq (-a)^2$
- $-3.2^2 = -3 \times 4 = -12$ and not $(-6)^2 = 36$
- $(2a)^2 = 4a^2 \neq 2a^2$
- $(-2x^2)^3 = -8x^6$ and not $-6x^5$
- $(7-2)^2 = 5^2 = 25$ and not $7^2 - 2^2 = 49 - 4 = 45$

CAN YOU?

Simplify the following expressions, without a calculator:

1. $5^2 \times 5^3.5$
2. $\frac{5^3}{5^{-2}}$
3. $4(5a^0)^2$
4. 3.2^{-2}
5. $(3.2)^{-2}$
6. $\frac{2^a 2^m 2^p}{2}$
7. $\frac{a^{10}b^3}{a^3b}$
8. $[(-2)^3]^2$
9. $(4x^3y^2)^3$
10. $6a^{-2}b^3 \times (2ab)^3$
11. $\frac{-4ab^7c^0}{8a^4b^2c^{-2}}$
12. $\frac{(2x^7y^{\frac{1}{2}})^2}{2xy}$
13. $\frac{(ab^{-1})^4}{(a^{-1}b^{-1})^2}$
14. $\frac{a^{-1}+b^{-1}}{(ab)^{-1}}$

Answers:

1. 5^6
2. 5^5
3. 100
4. $\frac{3}{4}$
5. $\frac{1}{36}$
6. $2^{a+m+p-1}$
7. a^7b^2
8. 64
9. $64x^9y^6$
10. $48ab^6$
11. $-\frac{b^5c^2}{2a^3}$
12. $2x^{13}$
13. $\frac{a^6}{b^2}$
14. $a + b$

Lesson 4	Simplifying Exponential Expressions

Simplify the following expressions:

Example 1: $81^{\frac{3}{4}}$

$= (3^4)^{\frac{3}{4}}$

$= 3^{4\times\frac{3}{4}} = 3^3 = 27$

> Write 81 as the product of its **PRIME factors**.

Example 2: $\frac{2^x.3^{x-2}.6^x}{4^{x-1}.9^{x+1}}$

Solution: $= \frac{2^x.3^{x-2}.(2.3)^x}{(2^2)^{x-1}.(3^2)^{x+1}}$

> Write each composite base as the product of its **PRIME factors**. **Note the use of brackets** to ensure that ALL factors remain under the influence of the

$= \frac{2^x.3^{x-2}.2^x.3^x}{2^{2x-2}.3^{2x+2}}$

> Apply exponential laws 3 and 4 as well as the rules for multiplying a monomial with a polynomial (remember?)

$= 2^{x+x-(2x-2)}.3^{x-2+x-(2x+2)}$

$= 2^{x+x-2x+2}.3^{x-2+x-2x-2}$

> Now apply laws 1 and 2 on the powers with the same bases. Note how we write this using ONLY the 2 bases and the use of brackets again. Then simplify

$= 2^2.3^{-4}$

$= \frac{2^2}{3^4} = \frac{4}{81}$

Example 2: $\frac{3^{x+1}+3^{x-1}}{3^{x+2}-3^x} = \frac{3^x.3^1+3^x.3^{-1}}{3^x.3^2-3^x}$

> Note: we cannot apply the exponential laws since the numerator and denominator is not ONE TERM – so we must **FACTORISE**. Apply law 1 backwards and we have 3^x as CF

$= \frac{3^x(3^1+3^{-1})}{3^x(3^2-1)}$

$= \frac{3^1+3^{-1}}{3^2-1} = \frac{\frac{10}{3}}{8} = \frac{10}{3}\times\frac{1}{8} = \frac{5}{12}$

Example 3: $\frac{4^x-1}{2^x-1}$

$\frac{2^{2x}-1}{2^x-1} = \frac{(2^x)^2-1}{2^x-1}$

> $2^{2x}-1 = (2^x)^2 - 1$ leading to the difference of 2 squares to factorise!

$= \frac{(2^x-1)(2^x+1)}{(2^x-1)} = 2^x + 1$

CAN YOU?

Simplify the following expressions:

1. $64^{\frac{2}{3}}$
2. $\frac{1}{2}$ of 2^{50}
3. $\frac{(2^3.3^2)^2}{2^2.3^3}$
4. $\frac{(125x^3)^{\frac{1}{3}}}{(5x^{\frac{1}{2}})^2}$
5. $\frac{2^x.4^{x+1}.3^{3x-1}}{6^{3x-1}}$
6. $\frac{18^{x+1}}{2^{x+1}.9^{x+2}}$
7. $\frac{9^{n+1}+3^{2n-1}}{9^n}$
8. $\frac{3^{n+4}-6.3^{n+1}}{7.3^{n+2}}$
9. $\frac{2^{x+2}-2^{x+1}}{2^x+2^{x+2}}$
10. $\frac{5.2^x-4.2^{x-2}}{2^x-2^{x-1}}$
11. $\frac{4^x-25}{2^x+5}$
12. $\frac{4^x-2^x}{2^{x+1}-2}$
13. * $\frac{9^x-3^x-6}{3^x+2}$

Answers:

1. 16
2. 2^{49}
3. $2^4.3 = 48$
4. $\frac{1}{5}$
5. 8
6. $\frac{1}{9}$
7. $\frac{28}{3}$
8. 1
9. $\frac{2}{5}$
10. 8
11. $2^x - 5$
12. $\frac{2^x}{2} = 2^{x-1}$
13. $3^x - 3$

Lesson 5	Solving Exponential equations

Exponential equations where the unknown is in the exponent

Rule: If $a^x = a^k \Rightarrow x = k$ for equal powers: if the bases are the same, then the exponents are the same as well

Solve for x:

Make the bases the same; usually prime numbers as base

Example 1: $3^{x+2} = 27$

Solution: $3^{x+2} = 3^3$

Since bases are the same; exponents must be equal

$\therefore x + 2 = 3$

$\therefore x = 1$

Make bases the same. Note the use of brackets.

Example 2: $4^{x+1}.8^x = 32.2^{3x-1}$

$\therefore (2^2)^{x+1}.(2^3)^x = 2^5.2^{3x-1}$

Apply Law 1 to get one power both sides and apply Rule

$\therefore 2^{2x+2}.2^{3x} = 2^5.2^{3x-1}$

$\therefore 2^{5x+2} = 2^{3x+4}$

$\therefore 5x + 2 = 3x + 4$

$\therefore 2x = 2$

$\therefore x = 1$

LHS is not ONE TERM $\Rightarrow$ FACTORISE.

Example 3: $2^{x+2} - 2^x = \frac{3}{4}$

$\therefore 2^x.2^2 - 2^x = \frac{3}{4}$

$\therefore 2^x(2^2 - 1) = \frac{3}{4}$

$\therefore 2^x(3) = \frac{3}{4}$

$\therefore 2^x = \frac{3}{4} \times \frac{1}{3}$

$\therefore \mathbf{2^x} = \frac{1}{4} = \mathbf{2^{-2}}$

$\therefore x = -2$

Equations where the unknown is in the base

Example 5: $4x^{\frac{1}{4}} = 16$

Get the power alone on 1 side

$\therefore x^{\frac{1}{4}} = 4$

$\times$ the exponents both sides with the reciprocal of the exponent of x to solve

$\therefore \left(x^{\frac{1}{4}}\right)^{\frac{4}{1}} = (4)^4$

$\therefore x = 256$

CAN YOU?

Solve for x in the following equations:

1. $2^{x-4} = 32$
2. $3^{x-2}.9^x = \frac{1}{27}$
3. $2.3^{x+1} = 162$
4. $3^{x+1} - 3^{x-1} = 24$
5. $3.5^{x-1} + 4.5^x = \frac{23}{25}$
6. $x^{\frac{5}{3}} = 32$
7. $2x^{\frac{3}{4}} + 3 = 57$
8. $3^{x+1} - \left(\frac{1}{3}\right)^{-x-3} = -11 + 3^{x+2}$
9. $(3^x - 9)(3^x - 1) = 0$
10. $4 \times 3^{2x} = 9 \times 2^{2x}$

Answers:

1. 9
2. $-\frac{1}{3}$
3. 3
4. 2
5. -1
6. 8
7. 81
8. -1
9. 2 or 0
10. 1

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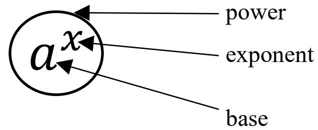
<tr><td>ACTIVITIES</td><td>Consider other exercises from your Mathematics Textbook</td></tr>
<tr><td>CONSOLIDATION</td><td>


Exponential laws:

1. $a^n \times a^m = a^{n+m}$
2. $\frac{a^n}{a^m} = a^{n-m}$
3. $(a^m)^n = a^{mn}$
4. $(ab)^n = a^n b^n$
5. $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
6. $a^0 = 1$
7. $a^{-n} = \frac{1}{a^n}$ and $\frac{1}{a^{-n}} = a^n$

Exponential equations:

If $a^x = a^k \Rightarrow x = k$
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<tr><td>VALUES</td><td>Dear learner. MATHEMATICS can OPEN DOORS for you, but only if you work hard at it. You will however, close that door on yourself through your unwillingness to practice every day. Keep up the good work!</td></tr>
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